

## Hydrological Drought Forecasting using ARIMA Models (Case Study: Karkheh Basin)

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**ABSTRACT** The present research was planned to evaluate the skill of linear stochastic models known as ARIMA and multiplicative Seasonal Autoregressive Integrated Moving Average (SARIMA) model in the quantitative forecasting of the Standard Runoff Index (SRI) in Karkheh Basin. To this end, SRI was computed in monthly and seasonal time scales in 10 hydrometric stations in 1974-75 to 2012-13 period of time and then the modeling of SRI time series was done to forecast the one to six months of lead-time and up to two seasons of lead-time. The SRI values related to 1974-75 to 1999-2000 were used to develop the model and the residual data (2000-2001 to 2012-13) were used in model validation. In the validation stage, the observed and the predicted values of SRI were compared using correlation coefficient, error criteria and statistical tests. Finally, models skills were determined in view point of forecasting of lead-time and the time scale of drought evaluation. Results showed that the model accuracy in forecasting two months and one season of lead-time was high. In terms of the forecasting of SRI values, the skill of SARIMA in monthly time scale (with a RMSE and a MAE of 0.61 and 0.45 respectively and a correlation coefficient average of 0.72) was better than its skill in seasonal time scale. The application of SARIMA in monthly time scale was therefore preferred to its application in seasonal time scale.

**Key words:** Drought duration, Drought magnitude, Standardized runoff index, Stochastic models, Time series models

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## 1 INTRODUCTION

Drought is one of the inseparable characteristics of each climate that is occurred due to the long term lack of precipitation (Morid *et al.*, 2006; Bazrafshan and Khalili, 2013; Azarakhshi *et al.*, 2013). This physical phenomenon in long term leads to the agricultural drought and then the hydrological drought and decreases the water resources through declining the surface and groundwater flows (Liu and Hwang, 2015). Forecasting the time of drought occurrence plays an important role in planning and management of natural resources and water resources systems in a watershed scale (Jalal kamali *et al.*, 2015).

Efficiency of drought monitoring system is affected by an index which is selected regarding the drought condition in the region. Over the years, diverse indices have been innovated to monitor the drought in meteorology, agriculture, hydrologic, and social-economic parts (Mendicino *et al.*, 2008) that each index anyhow reflects the related characteristics (American Meteorological Society, 1997). Among the diverse indices to monitor the climatic drought, SPI (McKee 1993) as the most famous index, is used extensively in all of the world in terms of the simple access to the data (precipitation); also, the possibility of its calculation in each time scale; the possibility of calculating the magnitude, frequency, and duration of drought; the possibility of early diagnosis of soil moisture and the possibility of showing the spatial distribution of areas under drought (Hayes 1999; Mishra and Desai, 2005).

Existing indices for characterizing a hydrological drought such as Surface Water Supply Index (SWSI) (Shafer and Dezman, 1982; Garen, 1993; or Palmer Hydrological Drought Index (PHDSI) (Alley, 1984; Karl and Knight, 1985; Karl, 1986) and Reconnaissance Drought Index (RDI) (Tsakiris and Vangelis, 2005; Nalbantis and Tsakiris, 2009; Bazrafshan

*et al.*, 2010) are data demanding and computationally intensive. On the other hands, for monitoring meteorological drought have been proposed very simple and effective indices such as Standardized Precipitation Index (SPI) (McKee 1993 and 1995). An index similar to the SPT based on the monthly average streamflow which is named Standard Runoff Index (SRI) was used to solve this problem. This index was the first time suggested by Ben-Zvi (1987) and then was developed by Modares (2006), Shukla and wood (2008), Nalbantis and Tsakiris (2009), Lorenzo-Lacruz *et al.* (2012) and Hosseinzadeh Talaei *et al.* (2014).

The time series models used in the streamflow forecasting process are mostly linear models. They were built under the assumption that the process follows normal distribution, but most streamflow processes are nonlinear (Wang, 2006). The stochastic models was classified in to two categories of (1) linear models as the auto-regressive models (AR), moving average models (MA), auto-regressive moving average models, (ARMA) (Box and Jenkins, 1976), and disaggregation models (Salas 1988); and (2) non-linear models as the Fractional Gaussian Noise models, FGN (Mandelbrot and Van Ness, 1968), the broken line models, BL (Rodríguez-Iturbe 1972).

Mishra and Desai (2005 and 2006) and Fathabadi *et al.* (2009) focused on drought forecasting using SPI as a drought indicator and ARIMA model. The predicted results using the best models were compared with the observed data. The predicted value decreases with increase in lead-time. Abudu *et al.* (2010) predicted the drought using seasonal autoregressive integrated moving average (SARIMA) and autoregressive integrated moving average (ARIMA) models in Kizil River in China. Results revealed the appropriate skill of the model in forecasting the one month ahead. In addition, Han *et al.* (2010) and Lorenzo- Lacruz *et al.* (2012) believed that time

series model had the skill of maximum 3 lead-time ahead. Tabari *et al.* (2013) also developed hydrological droughts monitoring by using the streamflow drought index (SDI) in the mountainous regions of northwestern Iran. Results showed that almost all the stations experienced extreme the driest years during the examined period 12 years. Hejabi *et al.* (2013) in their research resulted that the skill of ARIMA model decreases by increasing the lead-time of forecast. Alam *et al.* (2014) predicted the climatologically drought in 3, 6, 9, 12 and 24 time scales by ARIMA and SARIMA. Results showed a good agreement between the observed data and the forecasted data up to 3 lead-time ahead. Ultimately, Wang *et al.* (2015) predicted the annual flow by ARIMA. Based on the results, skill of the model increased considerably in combination with the Artificial Neural Network (ANN).

The ARIMA models seem to offer a potential to develop reliable forecasts towards prediction of drought duration and severity. The ARIMA model approach has several advantages over other methods, in particular, its forecasting capability, its richer information on time-related changes, or the consideration of serial correlation between observations. Also, few parameters are required for describing time series, which exhibit non-stationary both within and across the seasons.

The aim of this research is assessing the efficiency of stochastic known as (ARIMA) and (SARIMA) models in forecasting the hydrologic drought in monthly and seasonal time scales and determining the amount of the efficiency of the models in the forecasting lead-time.

## 2 MATERIALS AND METHODS

### 2.1 Study area

The study area, Karkheh Basin in west of the Iran, located in the central and southern regions of the Zagros Mountain range and its area is

more than 50000 km<sup>2</sup>. In terms of the geographical coordination, this region has been extended between 46° 06' - 49° 10' E longitudes and 30° 58' - 34° 56' N latitudes (Figure 1). Hydrologically, the basin is divided into five sub-basins viz. Gamasiab, Qarasou, Kashkan, Saymareh and south Karkheh. Water in the basin is mainly used for agriculture production, domestic supplies, and fish farming but also serves to sustain the environment. For the latter, a major concern is the sustainability of the Hoor-Al-Azim swamp that is a Ramsar site located at the Iran-Iraq border (Karimi and Shahedi, 2013).

Among the stations located in five principle sub basins of Karkheh Basin, 10 hydrometric stations with 38 years statistical period length from 1974-75 to 2012-13 were selected according to the appropriate spatial distribution and having sufficient data (Table 1). The average annual discharge changes from 3.3 to 86 m<sup>3</sup> s<sup>-1</sup>. The maximum discharge is 190.6 which is related to the Pol-E Zal Station in the outlet of the river basin and the minimum discharge is 0.7 m<sup>3</sup> s<sup>-1</sup> which is related to the Doabmerk Station. The highest amount of the standard deviation is in the Pol-E Zal Station.

### 2.2 Standardized Runoff Index (SRI) for hydrologic drought analysis

Based on the computational principles of SRI, at first, the monthly discharge amounts are fitted to an appropriate distribution. Researches have shown that gamma distribution and log normal or bivariate log normal distributions had the best fitting in small and large basins, respectively (Nalbantis and Tsakiris, 2009). Therefore, monthly discharge amounts were fitted by the relation of the selected distribution and the cumulative probability of the selected distribution was computed. Accordingly transformation of cumulative co-probability of selected distribution to the normal distribution was done. In the last phase, normalized standardized Z variable or SRI related to each

amount of discharge in each station was extracted from normal cumulative probabilities curve (Shukla and Wood, 2008; Nalbantis and

Tsakiris, 2009). SRI classification has been presented in Table 2.

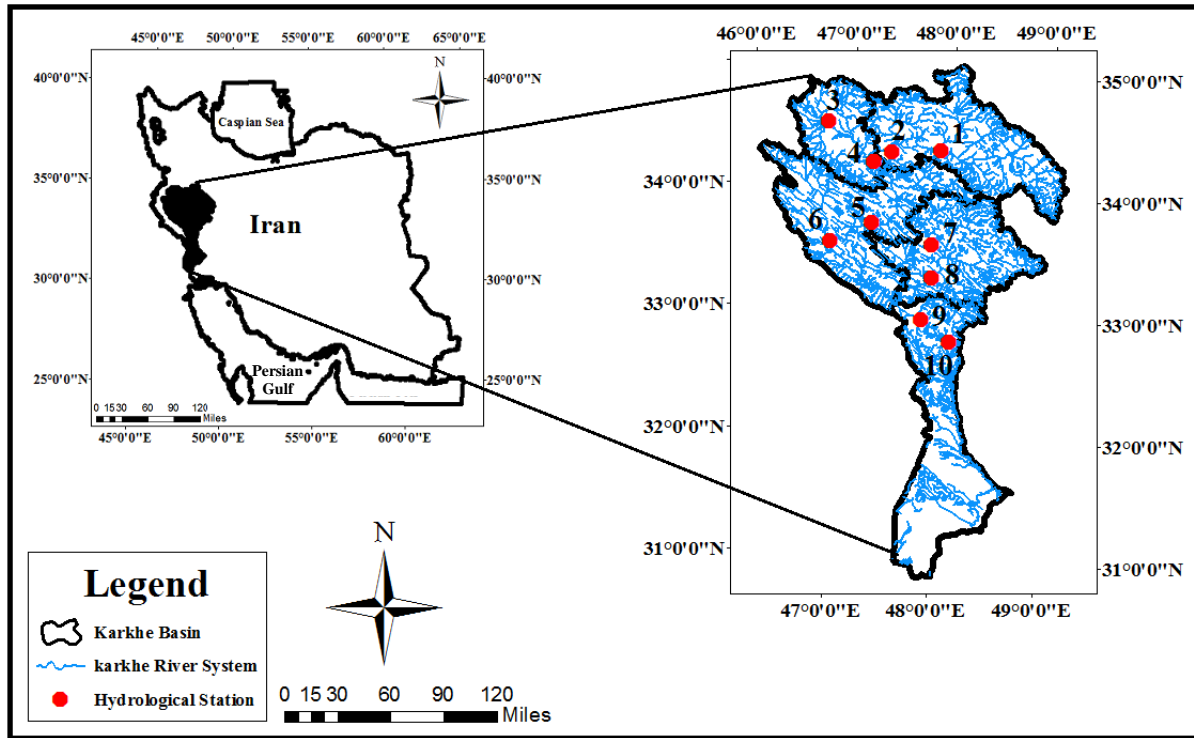


Figure 1 Spatial distribution of the hydrometric stations in Karkheh Basin, Iran

Table 1 Characteristics of selected hydrometric stations in Karkheh Basin, Iran

| Station Code | Sub basin     | Hydrometric Station | Latitude | Longitude | Statistical properties of annual discharge series (1974–2012) |                                       |                                       |                    |
|--------------|---------------|---------------------|----------|-----------|---|---------------------------------------|---------------------------------------|--------------------|
|              |               |                     |          |           | Average   | Max (m <sup>3</sup> s <sup>-1</sup> ) | Min (m <sup>3</sup> s <sup>-1</sup> ) | Standard Deviation |
| 1            | Gamasiab      | Doab                | 47° 54'  | 34° 22'   | 15.42   | 32.44                                 | 4.06                                  | 7.66               |
| 2            | Gamasiab      | Polchehr            | 47° 26'  | 34° 20'   | 32.5  | 77.3                                  | 6.2                                   | 15.2               |
| 3            | Gharesoo      | Doabmerk            | 47° 46'  | 34° 33'   | 5.5   | 12.06                                 | 0.71                                  | 2.91               |
| 4            | Gharesoo      | Ghourbaghestan      | 47° 15'  | 34° 13'   | 20.6  | 4.6                                   | 3.3                                   | 9.4                |
| 5            | Kashkan       | Holilan             | 47° 15'  | 34° 44'   | 71.57   | 146.47                                | 19.16                                 | 22.29              |
| 6            | Kashkan       | Tangesazoo          | 46° 50'  | 34° 33'   | 3.9   | 8.4                                   | 1.2                                   | 1.6                |
| 7            | Seimareh      | Visan               | 47° 57'  | 34° 29'   | 10.9  | 19.16                                 | 5.72                                  | 3.57               |
| 8            | Seimareh      | Afarineh            | 47° 53'  | 34° 19'   | 3.3   | 4.4                                   | 1.28                                  | 0.52               |
| 9            | Karkheh Paeen | Jelogir             | 47° 48'  | 32° 58'   | 9.36  | 18.76                                 | 3.17                                  | 3.65               |
| 10           | Karkheh Paeen | Pol-Ezal            | 47° 10'  | 32° 25'   | 86  | 190.6                                 | 14.4                                  | 36.5               |

**Table 2** Hydrological drought classification by SRI value and corresponding event probability (Nalbantis and Tsakiris, 2009; Hosseinzadeh Talaei *et al.*, 2014)

| State | Description      | Criterion            |
|-------|------------------|----------------------|
| 0     | Non- drought     | $SRI \leq 0$         |
| 1     | Mid-drought      | $-1 \geq SRI < 0$    |
| 2     | Moderate drought | $-1.5 \geq SRI < -1$ |
| 3     | Severe drought   | $-2 \geq SRI < -1.5$ |
| 4     | Extreme drought  | $SRI < -2$           |

**2.3 Time series models**

Linear stochastic models known as ARIMA are used in the present study. AR models have been extensively used in hydrology and modeling in annual time scale for water resource. Autoregressive- moving average mixed behavior could be modeled by adding moving average (MA) component to the Autoregressive (AR) component. An AR model of order p and moving average model of order q combined to obtain the mixed ARMA of order (p,q) (Jalal Kamali, 2015). It's defined by (Eq. 1):

$$Z_t = \sum_{i=1}^p \phi_i Z_{t-i} - \sum_{j=0}^q \theta_j \varepsilon_{t-j} \quad \text{for } \theta_0 = -1 \quad (1)$$

Where  $Z_t$  is the observed series,  $\phi$  is the polynomial of order p and  $\theta$  is the polynomial of order q.

AR, MA and ARMA can be used when the data are stationary. ARMA models can be extended to non-stationary series by allowing differencing of data series. These models are called ARIMA models.

The general non-seasonal ARIMA model is AR to order p and MA to order q and operates on the  $d^{th}$  difference of the time series  $Z_t$ ; thus, a model of the ARIMA family is classified by three parameters (p, d, q) that can have zero or positive integral values (Mishra and Desai, 2005; Fernandez, 2009).

The general non-seasonal ARIMA model can be written following based on (Eq. 2):

$$\phi(B)(1-B)^d Z_t = \theta(B)\varepsilon_t \quad (2)$$

Where  $\phi$  and  $\theta$  are polynomials of order p and q, respectively (Eqs. 3 and 4):

$$\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \quad (3)$$

$$\theta(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) \quad (4)$$

Box and Jenkins (1979) generalized ARIMA (p, d, q)<sub>x</sub> and obtained the multiplicative ARIMA (p, d, q)<sub>x</sub> (P, D, Q)<sub>w</sub> model which consist of seasonal ARMA (P, Q) fitted to the  $D^{th}$  seasonal difference of the data coupled with an ARMA (p, q) model fitted to the  $d^{th}$  difference of the residual of the former model (Eq. 5):

$$\Phi_p(B^w)\phi_p(B)(1-B^w)^D(1-B)^d Z_t = \Theta_Q(B^w)\theta_q(B)\varepsilon_t \quad (5)$$

Where p is the order of non-seasonal auto regression, d the number of regular differencing, q the order of non-seasonal MA, P the order of seasonal auto regression, D the number of seasonal differencing, Q the order of seasonal MA, w is the length of season  $\Phi_p$  and  $\Theta_Q$  are seasonal polynomials of order P and Q (Jalal Kamali 2015).

Time series model development modeling consists of three stages identification, estimation, and diagnostic test (Box and Jenkins, 1976; Mishra and Desai, 2005; Modarres, 2006; Duru, 2010; Wang 2015). The identification stage involves transforming the data to the normality.

Box and Jenkins (1976) described the model identification step as a rough procedure for laying down the initial model structure. This stage identified by examining autocorrelation function

(ACF), partial autocorrelation function (PACF) (Duru 2010; Wang *et al.* 2015) and Akaike information criterion (AIC) (Akaike, 1987), corrected Akaike information criterion (AICC) and Bayesian information criterion (SBC). Minitab software Version 17 and XLSTAT 2015 were used for time series model development in this stage. ACF and PACF were used to statistically measure if earlier values in the series have some relation to later values. By looking at the ACF and PACF plots of the differenced series, we could tentatively identify the numbers of AR and/or MA terms that were needed.

The model gives the minimum (AICC) and (SBC) which is selected as the best-fit model (Mishra and Desai, 2005; Duru, 2010). The mathematical formulation for the AICC and SBC (Schwarz 1978) was developed as following (Eqs. 6 and 7):

$$AICC(p, q, P, Q) = N \ln(\sigma^2_{(\varepsilon)}) + \frac{2(p+q+P+Q+1)N}{(N-p-q-P-Q-2)} \quad (6)$$

$$SBC = -2 \log(L) + (p + q + P + +Q) \ln(N) \quad (7)$$

Where N denotes the number of observations, L denotes the likelihood function of the ARIMA models and it is a monotonically decreasing function of the sum of squared residuals.

SBC is usually a better criterion than AIC when the number of samples is low. AICC is the revised version of AIC and acts well even by low number of samples (Mishra and Desai, 2005; Alam 2014).

After the identification of model using the AICC and SBC criteria estimation of parameters is done with Minitab 17 software. After identification of the model and estimation of the parameters, diagnostic test is applied to the fitted model to verify the adequacy of the model. Several tests are employed for diagnostic test that consists of: Portmanteau lack-of-fit test, Normal probability plot of residuals and Kolmogorov–

Smirnov statistics of residuals and White noise ACF and PACF of residuals.

### 2.3.1 Portmanteau lack-of-fit test to check the independence of residuals.

Portmanteau lack-of-fit test was modified to Ljung-Box-Pierce statistics proposed by Ljung and Box (1978) employed to check the independence of residuals. In order to test the null hypothesis that a current set of autocorrelations is white noise, test statistics are calculated for different total numbers of successive lagged autocorrelations using the Ljung-Box-Pierce corrected statistics (Q\*r test) to test the adequacy of the model. The Q\*r statistic is formulated as follows Eq. 8: (Duru, 2010; Lee and Ko, 2011).

$$Q^* = n(n+2) \sum_{k=1}^L \frac{r_k^2(\varepsilon)}{n-k} \quad (8)$$

Where L is the total number of lagged autocorrelations under investigation,  $r_k$  is the sample, and autocorrelation of the residuals at lag k.  $Q_r^*$  values are compared with the value of  $\chi^2$  distribution with a degree of freedom and a significant level of 95%, N is total observation.

### 2.3.2 Normal probability plot of residuals and Kolmogorov–Smirnov statistics of residuals

Kolmogorov-Smirnov test (K-S test) was used to test the normality of residuals from different sets of models of the fit of data. (Eq. 9):

$$D = \max |F(x) - \hat{F}(x)| \quad (9)$$

Where D is the maximum deviation,  $F(x)$  the completely specified theoretical cumulative distribution function under the null hypothesis,  $\hat{F}(x)$  is the sample cumulative density function based on n observations. For a chosen significance level  $\alpha$ , for D greater than the critical

value  $D_{tab}$ , the null hypothesis related to normality is rejected for the chosen level of significance.

Also, basic statistical properties are compared between observed and forecasted data for 6 month and 2 seasons lead-time, using Z-test for the means and F-test for standard deviation (Haan, 1977).

The Z test was used to compare the average of the observed and forecasted values. If we select  $n$  random samples of normal community with a  $\mu$  average and a  $\sigma$  standard deviation, the  $\bar{x}$  average will be distributed normally with a  $\mu$  average and a  $\frac{\sigma}{\sqrt{n}}$  standard deviation. Therefore, the Z value was acquired by relation 10 (Eq. 10): (Montgomery, 2009).

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad (10)$$

For Z amounts less than 1.96, null hypothesis based on the equality of the forecasted and observed values is accepted.

The F test was used to compare the variance of forecasted values with the variance of the observed values. With  $n_1$  samples of the first community and  $n_2$  samples of the second community with variances  $s_1^2$  and  $s_2^2$  respectively, the F distribution with  $k_1$  and  $k_2$  degree of freedom is acquired by Eq. 11: if the values of computational F be less than the F of the table in 5% error level, null hypothesis based on the equality of the variances of the forecasted and observed values will be accepted (Eq. 11): (Montgomery, 2009).

$$\frac{S_1^2}{S_2^2} = F \quad (11)$$

### 2.3.3 White noise ACF and PACF of residuals

For a good forecasting model, the residuals, left over after fitting the model, must satisfy the

requirements of a white noise process. There are two useful applications related to PACF for the independence of residuals. The first is the correlogram drawn by plotting  $r_k$  against lag  $k$ , where  $r_k$  is the residual ACF function (Lee and Ko, 2011).

### 2.3.4 Model validation

Following tests were used to evaluate the accuracy of the forecasted standardized stream flow:

- Correlation between observed and forecasted time series (Eq. 12). The coefficient of correlation  $R$  was selected as the degree of collinearity criterion of level prediction (Wang 2015).
- The root mean square error for forecasted standardized stream flow (Eq. 13). (Roughani *et al.*, 2007; Shalamu *et al.*, 2011; Liu and Hwang, 2015)
- The mean absolute error (Eq. 14) (Mishra and Desai, 2006)

Each of the error coefficients provided special and unique information that the other coefficient don't provide them. RMSE showed the errors more than 1 as the overestimation and the errors less than 1 as the underestimation that this characteristic was so suitable to examine the terminal errors, namely the values of errors which were distributed at the two ends of the error distribution. Although, MAE coefficient considered the average of the real values of the error, it ignored the direction of the error variations (Makridakis *et al.* 2003). Therefore, those two mentioned indices at the above were used in this research to determine the error of the model in two seasonal and annual scales (Eqs. 12 to 14):

$$R = \frac{\frac{1}{N} \sum_{i=1}^N (X_O(i) - \bar{X}_O)(X_f(i) - \bar{X}_f)}{\sqrt{\frac{1}{N} \sum_{i=1}^N (X_O(i) - \bar{X}_O)^2} \sqrt{\frac{1}{N} \sum_{i=1}^N (X_f(i) - \bar{X}_f)^2}} \quad (12)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_f(i) - X_O(i))^2} \quad (13)$$

$$MAE = \frac{\sum_{i=1}^N |X_f(i) - X_o(i)|}{N} \quad (14)$$

Where  $X_o(i)$  and  $X_f(i)$  were respectively, the observed and forecasted SRI,  $\bar{X}_o$  and  $\bar{X}_f$  denoted their means, and N was the number data points considered.

### 3 RESULTS AND DISCUSSIONS

#### 3.1 Hydrological drought properties based on SRI

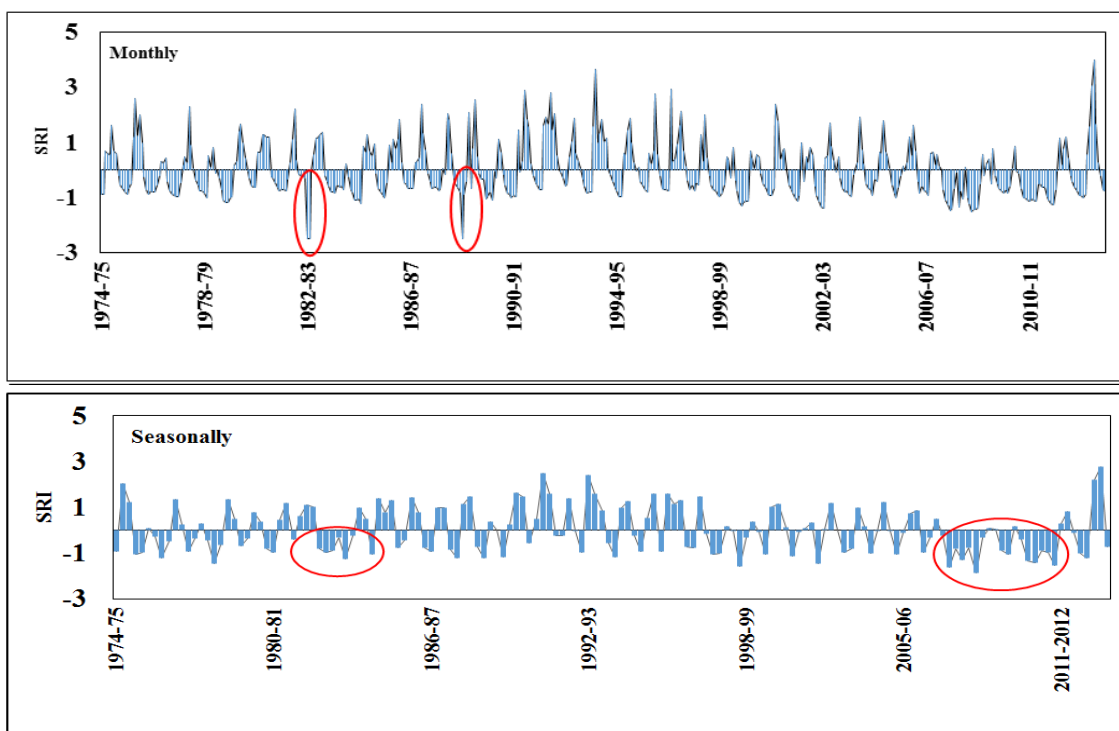
Based on the methodology of Shukla and Wood, 2008 and Nalbantis and Tsakiris, 2009, log normal distribution was identified as the most appropriate distribution to discharge data of the hydrometric stations of Karkheh Basin. Figure 2 showed the SRI values in monthly and seasonal scales. Based on this figure, there are

continuously wet and drought periods in the river basin. So that, the extremely drought is related to 1982-83 and 1988-89 years with a magnitude of -2.5 and duration of 3-month.

The regional time series of SRI value is calculated using the discharge weighted average over the Karkheh Basin. The time series of monthly and seasonal hydrologic drought are shown in Figure 2.

#### 3.2 Stochastic model development

The data set from 1974 -1975 to 2011-2012 were used to develop the model (water years). A split sample procedure was used for the calibration and validation of the model. In each of the monthly and seasonal database, the flow data from 1974-75 to 1999-2000 were used for calibration and the data from 2000-01 to 2012-13 were used for the validation of the model.



**Figure 2** SRI Monthly (Top) and seasonal (Bottom) time series based on the discharge weighted average over the Karkheh Basin



**3.2.1 Identification**

Identification of the general form model involves two steps. First, the data series is analyzed for stationary and normality. Second, the temporal correlation structures of the transformed data are identified by examining its autocorrelation (ACF) and partial autocorrelation (PACF) function (Ghanbarpour *et al.*, 2010).

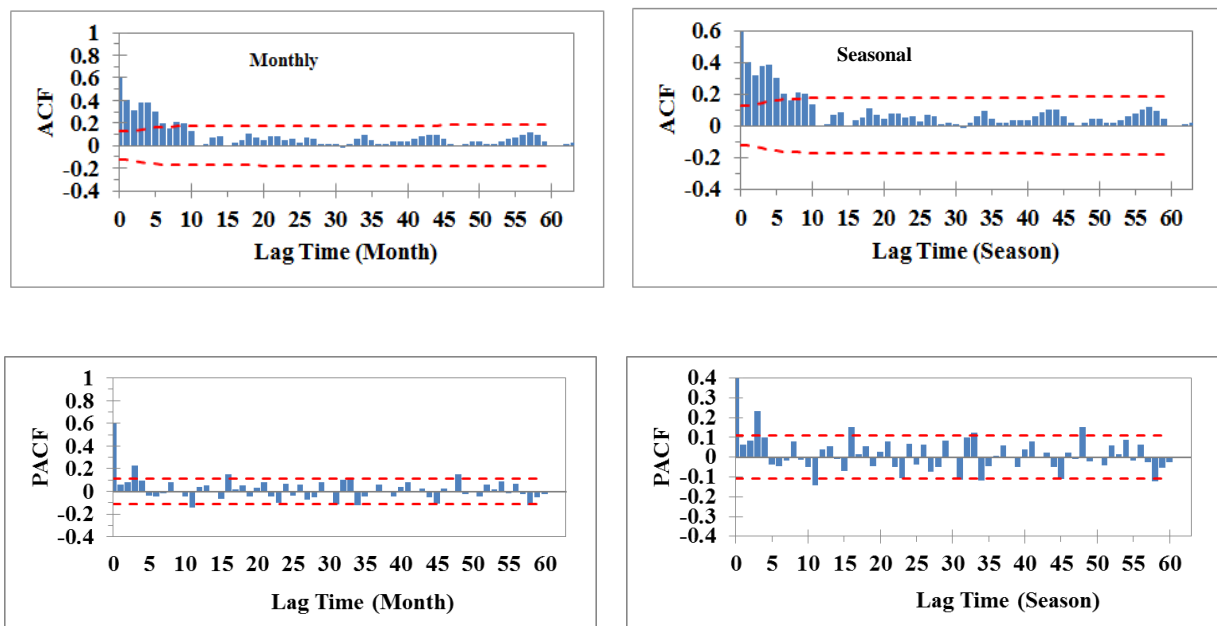
The ACF and PACF were estimated for monthly and seasonal SRI in the Visan hydrometric station which has been shown in Figure 3. The PACF showed that the series were stationary. The ACF was damping out in sine-wave manner with significant spikes at the first five lags. The first two values were

significant in PACF which indicated the process could be modeled as a combination of both AR and MA processes.

The identification of the best seasonal and non-seasonal models for hydrological drought in the different locations depends on the minimum AICC and SBC criteria which were presented in Table 3.

**3.2.2 Estimation**

After the identification of model using the AICC and SBC criteria, estimation of parameters is done. The summary of the statistical parameters of the best fitted models under the study have been given in Table 4.



**Figure 3** ACF (Top) and PACF (Bottom) plots used for stochastic monthly and seasonal series at Visan hydrometric station

**Table 3** Summary of AICC and SBC parameters of the best fitted ARIMA models

| Station Code | Model (Monthly)                    | AICC    | SBC     | Model (Seasonal)                  | AICC    | SBC     |
|--------------|------------------------------------|---------|---------|-----------------------------------|---------|---------|
| 1            | ARIMA (2,1,1)(1,0,0) <sub>12</sub> | 476.191 | 494.890 | ARIMA (0,1,3)(0,0,2) <sub>4</sub> | 245.858 | 261.055 |
|              | ARIMA (0,1,4)(0,0,3) <sub>12</sub> | 478.339 | 508.102 | ARIMA (0,1,3)(1,0,2) <sub>4</sub> | 247.813 | 265.391 |
|              | ARIMA (2,1,1)(2,0,0) <sub>12</sub> | 477.883 | 500.283 | ARIMA (1,1,1)(0,0,2) <sub>4</sub> | 247.601 | 260.371 |
| 2            | ARIMA (3,1,1)(2,0,1) <sub>12</sub> | 426.036 | 463.074 | ARIMA (1,1,1)(0,0,3) <sub>4</sub> | 247.605 | 262.737 |
|              | ARIMA (3,1,4)(2,0,1) <sub>12</sub> | 428.234 | 428.234 | ARIMA (0,1,2)(0,0,3) <sub>4</sub> | 247.823 | 262.955 |
|              | ARIMA (3,1,3)(2,0,1) <sub>12</sub> | 428.642 | 463.235 | ARIMA (1,1,1)(1,0,2) <sub>4</sub> | 248.697 | 263.829 |
| 3            | ARIMA (1,1,2)(1,0,0) <sub>12</sub> | 360.344 | 379.043 | ARIMA (1,1,3)(1,0,0) <sub>4</sub> | 210.002 | 225.199 |
|              | ARIMA (1,1,2)(2,0,0) <sub>12</sub> | 362.159 | 384.559 | ARIMA (0,1,3)(0,0,0)              | 213.367 | 223.666 |
|              | ARIMA (1,1,4)(1,0,0) <sub>12</sub> | 363.110 | 389.198 | ARIMA (013)(100) <sub>4</sub>     | 214.023 | 226.793 |
| 4            | ARIMA (1,1,1)(1,0,2) <sub>12</sub> | 495.018 | 517.418 | ARIMA (111)(100) <sub>4</sub>     | 244.060 | 254.359 |
|              | ARIMA (1,1,3)(1,0,2) <sub>12</sub> | 495.018 | 517.418 | ARIMA (111)(001) <sub>4</sub>     | 244.063 | 254.401 |
|              | ARIMA (1,1,3)(1,0,0) <sub>12</sub> | 499.541 | 514.526 | ARIMA (013)(001) <sub>4</sub>     | 244.532 | 257.302 |
| 5            | ARIMA (1,1,2)(0,0,1) <sub>12</sub> | 508.969 | 527.668 | ARIMA (1,0,0)(0,0,0)              | 267.339 | 272.589 |
|              | ARIMA (1,1,2)(1,0,0) <sub>12</sub> | 508.296 | 526.995 | ARIMA (1,1,1)(1,1,1) <sub>4</sub> | 269.933 | 277.719 |
|              | ARIMA (2,1,1)(1,0,0) <sub>12</sub> | 508.415 | 527.114 | ARIMA (0,1,3)(1,0,2) <sub>4</sub> | 270.392 | 287.970 |
| 6            | ARIMA (1,1,1)(0,0,1) <sub>12</sub> | 574.012 | 588.997 | ARIMA (0,1,2)(1,0,1) <sub>4</sub> | 270.350 | 283.120 |
|              | ARIMA (1,1,1)(1,0,0) <sub>12</sub> | 574.107 | 589.092 | ARIMA (0,1,3)(1,0,1) <sub>4</sub> | 272.092 | 287.289 |
|              | ARIMA (1,1,2)(1,0,0) <sub>12</sub> | 574.890 | 593.589 | ARIMA (1,1,1)(1,0,1) <sub>4</sub> | 272.087 | 284.857 |
| 7            | ARIMA (1,0,0)(0,1,2) <sub>12</sub> | 600.543 | 615.385 | ARIMA (1,1,1)(0,0,1) <sub>4</sub> | 280.628 | 290.928 |
|              | ARIMA (1,0,0)(1,1,1) <sub>12</sub> | 600.599 | 615.441 | ARIMA (1,1,1)(1,0,0) <sub>4</sub> | 280.726 | 290.95  |
|              | ARIMA (1,0,1)(0,1,2) <sub>12</sub> | 602.167 | 620.686 | ARIMA (1,1,2)(0,0,1) <sub>4</sub> | 282.386 | 295.156 |
| 8            | ARIMA (1,1,2)(1,0,3) <sub>12</sub> | 493.967 | 523.730 | ARIMA (0,1,2)(0,0,3) <sub>4</sub> | 269.916 | 285.113 |
|              | ARIMA (1,1,2)(1,0,4) <sub>12</sub> | 494.108 | 527.532 | ARIMA (0,1,3)(1,0,1) <sub>4</sub> | 271.486 | 286.683 |
|              | ARIMA (1,1,2)(2,0,3) <sub>12</sub> | 496.248 | 529.672 | ARIMA (0,1,3)(1,0,2) <sub>4</sub> | 271.915 | 289.494 |
| 9            | ARIMA(1,0,2) (0,0,0)               | 518.184 | 533.181 | ARIMA(1,1,1)(0,0,2) <sub>4</sub>  | 255.657 | 268.427 |
|              | ARIMA(2,1,2)(1,0,0) <sub>12</sub>  | 519.671 | 542.071 | ARIMA (1,1,1)(1,0,2) <sub>4</sub> | 256.819 | 272.016 |
|              | ARIMA(1,0,3) (0,0,0)               | 520.804 | 539.519 | ARIMA (0,1,2)(1,0,2) <sub>4</sub> | 257.694 | 272.891 |
| 10           | ARIMA (3,0,0) (0,0,0)              | 679.517 | 694.515 | ARIMA (1,1,1)(1,0,2) <sub>4</sub> | 289.539 | 304.736 |
|              | ARIMA (3,0,1) (0,0,0)              | 682.568 | 701.283 | ARIMA (1,1,2)(2,0,0) <sub>4</sub> | 294.837 | 310.034 |
|              | ARIMA (3,0,2) (0,0,0)              | 684.975 | 707.395 | ARIMA (1,1,2)(1,0,0) <sub>4</sub> | 293.927 | 306.697 |

**Table 4** Summary of the statistical analysis of model parameters

| Station Code | Model (Monthly)                        | parameter  | Variables in the model |                |          |        |
|--------------|--|------------|------------------------|----------------|----------|--------|
|              |  |            | Value of parameters    | Standard error | t- ratio | P<0.05 |
| 1            | ARIMA (2,1,1)<br>(1,0,0) <sub>12</sub> | $\phi_1$   | -0.8769                | 0.075          | -11.69   | 0      |
|              |  | $\phi_2$   | -0.0088                | 0.0583         | -0.15    | 0.88   |
|              |  | $\Phi_1$   | -0.1085                | 0.0575         | -1.89    | 0.06   |
|              |  | $\theta_1$ | -0.9251                | 0.0498         | -18.58   | 0      |
|              |  | $\phi_1$   | 0.2982                 | 0.1994         | 1.5      | 0.136  |
|              |  | $\Phi_2$   | -0.2902                | 0.1797         | -1.61    | 0.107  |
|              |  | $\Phi_3$   | 0.3918                 | 0.1288         | 3.04     | 0.003  |
| 2            | ARIMA (3,1,3)<br>(2,0,1) <sub>12</sub> | $\theta_1$ | -0.7142                | 0.3607         | -1.98    | 0.049  |
|              |  | $\theta_2$ | 0.0204                 | 0.0895         | 0.23     | 0.82   |
|              |  | $\theta_3$ | 0.2564                 | 0.1894         | 1.35     | 0.177  |
|              |  | $\Phi_1$   | -0.0052                | 0.1617         | -0.03    | 0.974  |
|              |  | $\Phi_2$   | 0.598                  | 0.1184         | 5.05     | 0      |
|              |  | $\Theta_2$ | -0.8104                | 0.3588         | -2.26    | 0.025  |
|              |  | $\phi_1$   | 0.8083                 | 0.0398         | 20.28    | 0      |
| 3            | ARIMA (1,1,2)<br>(1,0,0) <sub>12</sub> | $\theta_1$ | 0.1834                 | 0.0578         | 3.17     | 0.002  |
|              |  | $\theta_2$ | 0.7803                 | 0.0016         | 498.05   | 0      |
|              |  | $\Phi_1$   | 0.2074                 | 0.0181         | 11.48    | 0      |
|              |  | $\phi_1$   | 0.7695                 | 0.0434         | 17.74    | 0      |
|              |  | $\Phi_1$   | 0.638                  | 0.1618         | 3.94     | 0      |
| 4            | ARIMA (1,1,1)<br>(1,0,2) <sub>12</sub> | $\theta_1$ | 0.9504                 | 0.0213         | 44.68    | 0      |
|              |  | $\Theta_1$ | 0.6815                 | 0.1685         | 4.05     | 0      |
|              |  | $\Theta_2$ | 0.1382                 | 0.0739         | 1.87     | 0.062  |
|              |  | $\phi_1$   | 0.7693                 | 0.0411         | 18.73    | 0      |
|              |  | $\theta_1$ | 0.7659                 | 0.0002         | 4689.55  | 0      |
| 5            | ARIMA (1,1,2)<br>(0,0,1) <sub>12</sub> | $\theta_2$ | 0.2232                 | 0.0159         | 14.02    | 0      |
|              |  | $\Theta_2$ | -0.077                 | 0.0576         | -1.34    | 0.183  |
|              |  | $\phi_1$   | 0.7387                 | 0.038          | 19.46    | 0      |
| 6            | ARIMA (1,1,1)<br>(0,0,1) <sub>12</sub> | $\theta_1$ | 0.9665                 | 0.0065         | 147.6    | 0      |
|              |  | $\Theta_1$ | 0.0564                 | 0.0564         | 1        | 0.319  |
|              |  | $\phi_1$   | 0.7467                 | 0.0337         | 22.16    | 0      |
| 7            | ARIMA (1,0,0)<br>(0,1,2) <sub>12</sub> | $\Theta_1$ | 0.7008                 | 0.0515         | 13.61    | 0      |
|              |  | $\Theta_2$ | 0.0401                 | 0.0529         | 0.76     | 0.449  |
|              |  | $\phi_1$   | 0.7467                 | 0.0337         | 22.16    | 0      |
|              |  | $\theta_1$ | 0.7008                 | 0.0515         | 13.61    | 0      |
| 8            | ARIMA (1,1,2)<br>(1,0,3) <sub>12</sub> | $\theta_2$ | 0.0401                 | 0.0529         | 0.76     | 0.449  |
|              |  | $\Theta_1$ | 0.7467                 | 0.0337         | 22.16    | 0      |
|              |  | $\Theta_2$ | 0.7008                 | 0.0515         | 13.61    | 0      |
|              |  | $\Theta_3$ | 0.0401                 | 0.0529         | 0.76     | 0.449  |
|              |  | $\phi_1$   | 0.8469                 | 0.0479         | 17.69    | 0      |
| 9            | ARIMA (1,0,2)                          | $\theta_1$ | -0.1091                | 0.0767         | -1.42    | 0.156  |
|              |  | $\theta_2$ | 0.201                  | 0.0723         | 2.78     | 0.006  |
|              |  | $\phi_1$   | 0.8162                 | 0.0548         | 14.89    | 0      |
| 10           | ARIMA (3,0,0)                          | $\phi_2$   | -0.2601                | 0.0698         | -3.72    | 0      |
|              |  | $\phi_3$   | 0.1995                 | 0.0548         | 3.64     | 0      |

### 3.2.3 Diagnostic test

Results of the normal probability (K-S test) of residuals and Portmanteau lack-of-fit test in monthly and seasonal scales in study stations have been presented in Table 5. Portmanteau test showed that the calculated value was less than the actual  $\chi^2$  value, which signified that the present models were adequate on the available data. K-S test showed that for all models the  $D_{cal}$  was less than  $D_{tab}$  at 5% significant level and satisfied that the residuals were normally distributed.

In order to determine whether the residuals of the selected models from the AFC and PACF graphs were independent several tests for diagnostic checks were used. The ACF and PACF of residuals for monthly and seasonal SRI time series in the Visan hydrometric station were demonstrated in Figure 4. Based on figure 4, ACF and PACF residuals in monthly and seasonal time scales were located in confidence range and the time independence test for residuals was accepted.

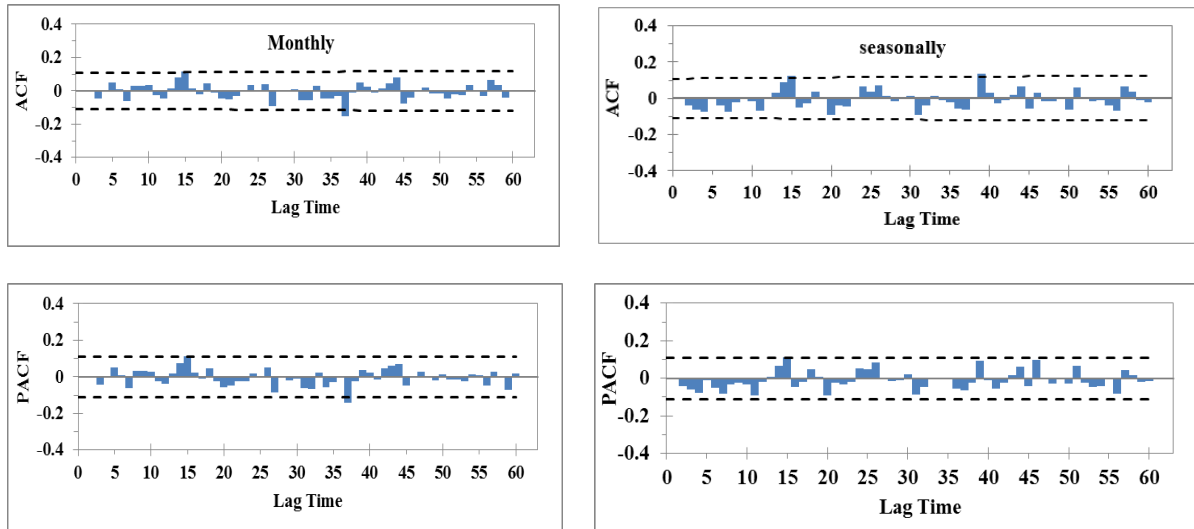
**Table 5** K-S test and  $Q_{r\text{ stat}}$  calculation of residuals for SRI series

| Station Code | SDI      | Model                                 | K-S Test  |           | Portmanteau Test |    |          |
|--------------|----------|---------------------------------------|-----------|-----------|------------------|----|----------|
|              |          |                                       | $D_{tab}$ | $D_{cal}$ | $Q^*$            | df | $\chi^2$ |
| 1            | Monthly  | ARIMA<br>(2,1,1)(1,0,0) <sub>12</sub> | 0.109     | 0.067     | 34.18            | 29 | 42.56    |
|              | seasonal | ARIMA (0,1,3)(0,0,2) <sub>4</sub>     | 0.095     | 0.072     | 5.63             | 6  | 59.12    |
| 2            | Monthly  | ARIMA<br>(3,1,3)(2,0,1) <sub>12</sub> | 0.109     | 0.064     | 22.08            | 23 | 35.17    |
|              | seasonal | ARIMA (1,1,3)(0,0,3) <sub>4</sub>     | 0.095     | 0.209     | 23.4             | 6  | 59.12    |
| 3            | Monthly  | ARIMA<br>(1,1,2)(1,0,0) <sub>12</sub> | 0.109     | 0.043     | 26.04            | 29 | 42.56    |
|              | seasonal | ARIMA (1,1,3)(1,0,0) <sub>4</sub>     | 0.095     | 0.058     | 12.56            | 6  | 59.12    |
| 4            | Monthly  | ARIMA<br>(1,1,1)(1,0,2) <sub>12</sub> | 0.109     | 0.040     | 26.56            | 28 | 34.41    |
|              | seasonal | ARIMA (1,1,1)(1,0,0) <sub>4</sub>     | 0.095     | 0.084     | 8.9              | 7  | 14.07    |
| 5            | Monthly  | ARIMA<br>(1,1,2)(0,0,1) <sub>12</sub> | 0.109     | 0.051     | 20.61            | 29 | 42.56    |
|              | seasonal | ARIMA (1,0,0)(0,0,0)                  | 0.095     | 0.063     | 11.86            | 10 | 18.31    |
| 6            | Monthly  | ARIMA<br>(1,1,1)(0,0,1) <sub>12</sub> | 0.109     | 0.081     | 29.34            | 30 | 43.77    |
|              | seasonal | ARIMA (0,1,2)(1,0,1) <sub>4</sub>     | 0.095     | 0.082     | 5.48             | 7  | 14.07    |
| 7            | Monthly  | ARIMA<br>(1,0,0)(0,1,2) <sub>12</sub> | 0.109     | 0.039     | 29.34            | 30 | 43.77    |
|              | seasonal | ARIMA (1,1,1)(0,0,1) <sub>4</sub>     | 0.095     | 0.073     | 1.40             | 9  | 16.92    |
| 8            | Monthly  | ARIMA<br>(1,1,2)(1,0,3) <sub>12</sub> | 0.109     | 0.067     | 26.27            | 26 | 38.89    |
|              | seasonal | ARIMA (0,1,2)(0,0,3) <sub>4</sub>     | 0.095     | 0.083     | 0.07             | 6  | 59.12    |
| 9            | Monthly  | ARIMA (1,0,2)                         | 0.109     | 0.071     | 34               | 30 | 43.77    |
|              | seasonal | ARIMA (1,1,1)(0,0,2) <sub>4</sub>     | 0.095     | 0.059     | 74.92            | 7  | 14.07    |
| 10           | Monthly  | AR (3,0,0)                            | 0.109     | 0.060     | 16.22            | 28 | 34.41    |
|              | seasonal | ARIMA (1,1,1)(1,0,2) <sub>4</sub>     | 0.095     | 0.092     | 12.13            | 7  | 14.07    |

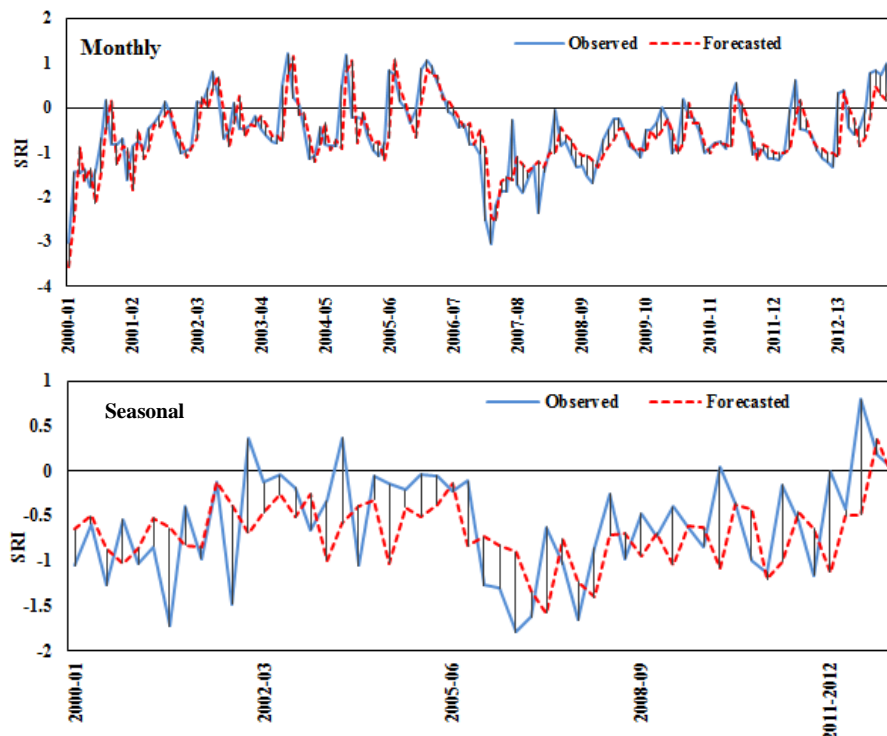
### 3.3 Monthly and seasonal hydrologic drought forecasting

The forecast was done for monthly and seasonal one lead-time using the best models from hydrometric data. The plot between observed

data and predicted data using the selected best model for all SRI time series is shown in Figure 5.



**Figure 4** Diagnostic test of the best-fitted ARIMA model for Visan hydrometric station SRI time series



**Figure 5** Comparison of observed data with forecasted data using best ARIMA models (in Visan hydrometric station)

Performance analysis of forecasting models consist of R, RMSE and MAE was represented in Table 6. Results showed that with a longer lead-time, the coefficient of correlation decreased and error coefficient increased between observed and predicted data. Therefore the selected best models from ARIMA building approach using a time series data of SRI can be used for the drought forecasting.

The Z test and F test computed for comparison of mean and variance, from each model were presented in Table 7. Result showed that there

was no significant difference between the mean and variance values of observed and predicted data. Since calculated Z values ( $Z_{cal}$ ) related to means were between  $Z_{critical}$  table values ( $\pm 1.96$  for two-tailed at a 5% significance level), the  $Z_{cal}$  values indicate that there was no significant difference between the mean values of observed and predicted data. Similarly, the  $F_{cal}$  values of standard deviation were smaller than the  $F_{critical}$  values at a 5% significance level. Thus, the results showed that predicted data preserved the basic statistical properties of the observed series.

**Table 6** Coefficient of correlation and error between observed and predicted data for different lead-time

| Station Code | Lag Time | Monthly |        |        |        |        |        | Seasonal |        |
|--------------|----------|---------|--------|--------|--------|--------|--------|----------|--------|
|              |          | 1       | 2      | 3      | 4      | 5      | 6      | 1        | 2      |
| 1            | R        | 0.766*  | 0.502* | 0.340* | 0.312  | 0.12   | 0.12   | 0.364*   | 0.086  |
|              | RMSE     | 0.503   | 0.533  | 0.690  | 0.736  | 0.781  | 1.00   | 0.644    | 0.731  |
|              | MAE      | 0.43    | 0.54   | 0.609  | 0.719  | 0.78   | 0.761  | 0.482    | 0.622  |
| 2            | R        | 0.79*   | 0.652* | 0.569* | 0.349* | 296*   | 0.276* | 0.277    | 0.132  |
|              | RMSE     | 0.502   | 0.754  | 0.802  | 0.819  | 0.9    | 0.690  | 0.74     | 0.776  |
|              | MAE      | 0.399   | 0.569  | 0.59   | 0.604  | 0.801  | 0.860  | 0.562    | 0.608  |
| 3            | R        | 0.701*  | 0.606* | 0.598* | 0.541* | 0.369* | 0.209* | 0.684*   | 0.302  |
|              | RMSE     | 0.56    | 0.68   | 0.698  | 0.742  | 0.801  | 0.9    | 0.620    | 0.838  |
|              | MAE      | 0.412   | 0.467  | 0.509  | 0.612  | 0.743  | 0.744  | 0.493    | 0.676  |
| 4            | R        | 0.77*   | 0.657* | 0.340* | 0.21   | 0.129  | 0.112  | 0.357*   | 0.135  |
|              | RMSE     | 0.509   | 0.619  | 0.698  | 0.788  | 0.802  | 0.823  | 0.707    | 0.739  |
|              | MAE      | 0.392   | 0.425  | 0.512  | 0.589  | 0.612  | 0.65   | 0.539    | 0.589  |
| 5            | R        | 0.743*  | 0.501* | 0.478* | 0.405* | 0.301* | 0.287* | 0.372*   | 0.164  |
|              | RMSE     | 0.75    | 1.29   | 1.45   | 1.76   | 1.89   | 1.99   | 0.759    | 0.788  |
|              | MAE      | 0.585   | 0.964  | 0.998  | 1.23   | 1.43   | 1.7    | 0.641    | 0.613  |
| 6            | R        | 0.720*  | 0.563* | 0.459* | 0.348* | 0.304* | 0.298* | 0.223    | 0.286  |
|              | RMSE     | 0.604   | 0.871  | 0.9    | 1.07   | 1.11   | 1.35   | 0.628    | 0.599  |
|              | MAE      | 0.359   | 0.645  | 0.98   | 1.00   | 1.05   | 1.19   | 0.526    | 0.523  |
| 7            | R        | 0.55*   | 0.412* | 0.387* | 0.345* | 0.319* | 0.3*   | 0.373*   | 0.147  |
|              | RMSE     | 0.98    | 0.976  | 0.98   | 0.988  | 0.989  | 0.977  | 0.805    | 0.833  |
|              | MAE      | 0.65    | 0.698  | 0.71   | 0.756  | 0.771  | 0.774  | 0.616    | 0.664  |
| 8            | R        | 0.798*  | 0.445* | 0.304* | 0.137  | 0.175  | 0.159  | 0.324*   | 0.315* |
|              | RMSE     | 0.60    | 0.96   | 1.05   | 1.01   | 1.075  | 1.12   | 0.959    | 0.923  |
|              | MAE      | 0.502   | 0.71   | 0.801  | .871   | .898   | 0.9    | 0.815    | 0.750  |
| 9            | R        | 0.789*  | 0.448* | 0.262* | 0.17   | -0.13  | -0.1   | 0.397*   | -0.02  |
|              | RMSE     | 0.504   | 0.723  | 0.743  | 0.904  | 0.90   | 0.96   | 0.908    | 0.981  |
|              | MAE      | 0.3     | 0.61   | 0.54   | 0.598  | 0.67   | 0.78   | 0.676    | 0.699  |
| 10           | R        | 0.634*  | 0.123  | -0.21  | -0.22  | 0.22   | -0.123 | -0.12    | -0.194 |
|              | RMSE     | 0.625   | 0.91   | 0.734  | 0.743  | 0.658  | 0.754  | 0.907    | 0.903  |
|              | MAE      | 0.501   | 0.70   | 0.702  | 0.7    | 0.677  | 0.70   | 0.753    | 0.772  |

**Table 7** Comparison of statistic properties of the observed and predicted data

| Station Code | SRI      | Variance observed | Variance forecasted | $F_{cal} < F_{tab}$ | Mean observed | Mean forecasted | $Z < 1.96$ |
|--------------|----------|-------------------|---------------------|---------------------|---------------|-----------------|------------|
| 1            | Monthly  | 0.565             | 0.574               | 0.98 < 1.76         | -0.70         | -0.716          | 0.132      |
|              | Seasonal | 0.44              | 0.19                | 0.38 < 1.94         | -0.710        | -0.796          | 0.99       |
| 2            | Monthly  | 0.76              | 0.645               | 0.1 < 1.76          | -0.245        | -0.515          | 0.245      |
|              | Seasonal | 0.567             | 0.180               | 0.308 < 1.94        | -0.494        | -0.492          | 0.16       |
| 3            | Monthly  | 0.801             | 0.603               | 1.32 < 1.7          | -0.834        | -0.723          | 0.23       |
|              | Seasonal | 0.714             | 0.44                | 0.62 < 1.94         | -0.868        | -0.966          | 0.83       |
| 4            | Monthly  | 0.513             | 0.414               | 1.23 < 1.7          | -0.47         | -0.71           | 0.81       |
|              | Seasonal | 0.533             | 0.239               | 0.44 < 1.94         | -0.529        | -0.558          | 1.45       |
| 5            | Monthly  | 0.441             | 0.792               | 0.552 < 1.7         | -0.314        | 0.1             | 0.91       |
|              | Seasonal | 0.346             | 0.44                | 0.42 < 1.94         | -0.357        | -0.229          | 1.2        |
| 6            | Monthly  | 0.644             | 0.406               | 1.58 < 1.7          | -0.534        | -0.876          | 0.094      |
|              | Seasonal | 0.626             | 0.282               | 0.45 < 1.94         | -0.647        | -0.684          | 0.202      |
| 7            | Monthly  | 1.1               | 0.9                 | 1.2 < 1.7           | -0.326        | -0.337          | 0.074      |
|              | Seasonal | 0.7               | 0.17                | 0.24 < 1.94         | -0.398        | -0.419          | 0.44       |
| 8            | Monthly  | 1.14              | 0.896               | 1.27 < 1.7          | -0.402        | -0.334          | 0.11       |
|              | Seasonal | 0.992             | 0.386               | 0.38 < 1.94         | -0.410        | -0.345          | 0.50       |
| 9            | Monthly  | 0.8               | 0.548               | 1.4 < 1.7           | 0.71          | 0.41            | 0.78       |
|              | Seasonal | 0.778             | 0.51                | 0.74 < 1.94         | -0.576        | -0.651          | 0.36       |
| 10           | Monthly  | 0.61              | 0.42                | 1.45 < 1.7          | -0.286        | -0.21           | 0.91       |
|              | Seasonal | 0.49              | 0.099               | 0.203 < 1.94        | -0.324        | -0.093          | 1.85       |

#### 4 CONCLUSION

The aim of this research is the evaluation of the skill of linear and multiplicative ARIMA models in forecasting the hydrologic drought in monthly and seasonal time scales in Karkheh Basin. In both time scales, correlation coefficient between observed and forecasted values decreased with increasing the lead - time and the value of error coefficients increase that this was compatible with the results of Mishra and Desai (2005 and 2006), Fathabadi *et al.* (2009), and Hejabi *et al.* (2013). ARIMA model, based on the previous time series memory, forecasts the values linearly and uses the previous forecast lead-time to forecast the next lead-time, therefore, the values of forecasting error increase cumulatively with increasing the lead-time and the values of correlation coefficient decreased.

Results of the ARIMA model in terms of the skill in the lead-time of forecast show that the model was able to predict the two months and one season of lead-time with a high accuracy that this was in agreement with the results of Abudu *et al.* (2010), Han *et al.* (2010), Lorenzo-Lacruz *et al.* (2012), Hejabi *et al.* (2013) and Alam *et al.* (2014).

Comparison of the results of ARIMA model in seasonal and monthly time scales showed that the ability of the model to forecast the drought in seasonal time scale was less than its ability in monthly time scale that its main cause was the low amount of data in seasonal time scale relative to the amount of data in monthly time scale. These results were exactly similar to the results of Ghanbarpour *et al.* (2010).

Time series models used in this research to forecast the 6 lead-time ahead in monthly time

scale and 2 lead-time ahead in seasonal time scale showed that multiplicative ARIMA model was able to predict the hydrologic drought with a high accuracy to at least the 3 lead-time. Therefore, if the forecasting was done in short term lead-time this model can be used to develop the programs combating with drought and sustainable management of water resources in the other watershed basins with similar hydro-climatic condition. Finally, use of the other forecasting tools like Wavelet Transforms, Artificial Neural Network (ANN), Support Vector Machine (SVM), and the other predictors such as climatic signals was suggested to forecast the hydrologic drought in medium and long term time scales.

## 5 REFERENCES

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### پیش‌بینی خشکسالی هیدرولوژیکی با استفاده از مدل‌های آریمای (مطالعه موردی: حوزه آبخیز کرخه)

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چکیده هدف از پژوهش پیش‌رو، بررسی کارایی مدل‌های تصادفی خطی و تصادفی مرکب در پیش‌بینی کمی شاخص جریان استاندارد شده (SRI) در حوزه آبخیز کرخه بوده است. برای این امر، محاسبه SRI، در مقیاس‌های زمانی ماهانه و فصلی، در ده ایستگاه هیدرومتری طی دوره ۱۹۷۵-۱۹۷۴ تا ۲۰۱۳-۲۰۱۲ انجام شد و مدل‌سازی سری‌های زمانی SRI برای پیش‌بینی یک تا شش گام به جلو در مقیاس ماهانه و تا دو گام به جلو در مقیاس فصلی انجام گرفت. مقادیر SRI مربوط به دوره ۱۹۷۵-۱۹۷۴ تا ۲۰۰۰-۱۹۹۹، برای توسعه مدل‌ها و مابقی (۲۰۰۱-۲۰۰۰ تا ۲۰۱۳-۲۰۱۲) برای صحت‌سنجی مدل مورد استفاده قرار گرفت. در مرحله صحت‌سنجی، مقایسه مقادیر مشاهده شده و پیش‌بینی شده SRI با استفاده از ضریب همبستگی، شاخص خطا و آزمون‌های آماری صورت پذیرفت. در نهایت، اولویت دقت مدل‌ها از دیدگاه‌هایی چون، افق زمانی پیش‌بینی و مقیاس زمانی بررسی خشکسالی تعیین شد. نتایج به‌دست آمده نشان داد: که دقت پیش‌بینی مدل در مقیاس زمانی ماهانه و فصلی به‌ترتیب دو ماه و یک فصل بعد بالا بوده است. همچنین، از نظر مهارت پیش‌بینی مقادیر SRI، مدل تصادفی مرکب در مقیاس زمانی ماهانه (به‌ترتیب با میانگین RMSE و MAE ۰/۶۱ و ۰/۴۵ و میانگین ضریب همبستگی ۰/۷۲) نسبت به مقیاس فصلی اولویت داشته است.

**کلمات کلیدی:** بزرگی خشکسالی، تداوم خشکسالی، شاخص استاندارد شده جریان، مدل‌های تصادفی، مدل‌های سری زمانی