

## Optimal Parameter Estimation for Nonlinear Muskingum Model based on Artificial Bee Colony Algorithm

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**ABSTRACT** Parameter estimation of the nonlinear Muskingum model is a highly nonlinear optimization problem. Although various techniques have been applied to optimize the coefficients of the nonlinear Muskingum flood routing models, but an efficient method for this purpose in the calibration process is still lacking. The accuracy of artificial bee colony (ABC) algorithm is investigated in this paper to optimize the coefficients of nonlinear Muskingum model. The performance of this algorithm was compared with other optimization techniques. For evaluating the ability of the ABC algorithm, several statistical criteria such as sum of the square error, sum of the absolute error, mean absolute error and mean relative error were used in the present study. ABC is an intelligent algorithm, which can effectively overcome the prematurity and slowed convergence speed of the traditional evolution algorithms. It determines the best parameter values in terms of the sum of square residual between the observed and routed outflows. The simulation results show that the performance of ABC algorithm with the sum of the square of the deviations between the computed and observed outflows (SSQ) of  $35.62 \text{ m}^3 \text{ s}^{-1}$ , the sum of the absolute value of the deviations between the computed and observed outflows coefficients (SAD) of  $23.2 \text{ m}^3 \text{ s}^{-1}$ , the mean absolute errors between the routed and observed outflows (MAE) of  $1.05 \text{ m}^3 \text{ s}^{-1}$  and the mean relative errors between the routed and observed outflows (MRE) of 2.9% is comparable to those of other algorithms. Thus ABC provided an efficient way for parameter optimization of the nonlinear Muskingum model.

**Key words:** Flood routing, Karoon River, Muskingum model, Parameters estimation, Parameters optimization

### 1 INTRODUCTION

The flood routing may be accomplished using two basic approaches that are generally called the "hydrologic routing approach" and the "hydraulic routing approach". The former is

based on the unsteady flow equations for open channels. The amount of time and effort required to calibrate the model increases with the degree of model sophistication. The hydrologic routing approach is based on the

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flow continuity principle and relation between discharge and storage. The Muskingum model, as one of the hydrologic routing techniques and the most frequently used flood routing methods in natural channels and rivers, was first developed by McCarthy (1938) after studies of the Muskingum river basin in Ohio (Chow *et al.*, 1988). The Muskingum model uses the flow continuity equation, which was written as:

$$\frac{dS_t}{dt} = I_t - O_t \quad (1)$$

$$S_t = K[xI_t + (1-x)O_t] \quad (2)$$

where:  $S_t$  is the channel storage at time  $t$ ;  $I_t$  and  $O_t$  are the rates of inflow and outflow at time  $t$ , respectively;  $K$  is the storage-time constant for the river reach, which has a value close to the flow travel time through the river reach; and  $x$  is weighting factor usually varying between 0 and 0.5 for reservoir storage, and between 0 and 0.3 for stream channels (Maidment, 1992). The values of the two parameters  $K$  and  $x$  are estimated using past records of inflow and outflow, and are then assumed to be characteristic values for a reach. The nonlinear form of the Muskingum model has been suggested by Gill (1978) and it is:

$$S_t = K[xI_t + (1-x)O_t]^m \quad (3)$$

where:  $m$  is additional parameter. This equation has more degrees of freedom than Eq. (2), which presumably would yield a closer fit to the nonlinear relation between storage and discharge. However, due to the presence of nonlinearity in the equation, the optimization procedure for estimating the optimal values of the parameters  $K$ ,  $x$ , and  $m$  becomes highly complex. The values of these parameters cannot easily be graphically derived from historical

inflow and outflow hydrographs. Therefore alternative parameter optimization techniques are intensively required that some of them have been reported for estimating parameters of the nonlinear Muskingum model (Gill, 1978; Tung, 1985; Aldama, 1990; Yoon and Padmanabhan, 1993; Mohan, 1997; Kim *et al.*, 2001; Das, 2004; Geem, 2006; Al-Humoud and Esen, 2006; Chen and Yang, 2007; Chu, 2009; Chu and Chang, 2009; Wang *et al.*, 2009; Luo and Xie, 2010; Wang *et al.*, 2010; Barati, 2011; Karahan, 2012, Karahan *et al.*, 2012, Gemm, 2014).

In the past years, some researchers adopted many optimization methods to optimize the parameters of the nonlinear Muskingum model. Mohan (1997) used genetic algorithm (GA) for parameter estimation of nonlinear Muskingum method and compared its performance with the NONLR procedure suggested by Yoon and Padmanabhan (1993). Kim *et al.* (2001) applied a harmony search (HS) algorithm for estimation of the three parameters of nonlinear Muskingum model. Das (2004) applied the Lagrange-multiplier technique to estimate the parameters for linear and nonlinear Muskingum models. The result obtained by the Lagrange multiplier was not as good as the results obtained by other researchers, who estimated the parameter values using the finite-difference form rather than the original hydrologic continuity equation. Geem (2006) introduced the Broyden-Fletcher-Goldfarb-Shanno (BFGS) technique. AL-Humoud and Esen (2006) presented two approximate methods for estimating Muskingum flood routing parameters. Chen and Yang (2007) developed the Gray-encoded accelerating genetic algorithm (GAGA) for parameter optimization of Muskingum model. Chu (2009) applied Adaptive Neuro-Fuzzy Inference System (ANFIS) and Muskingum model in flood routing. Chu and Chang (2009) applied particle swarm optimization (PSO) to estimate the

parameters of the nonlinear Muskingum model. Simulation results indicate that the proposed scheme can improve the accuracy of the Muskingum model for flood routing. Wang *et al.* (2009) proposed a hybrid chaotic genetic algorithm (HCGA) based on chaotic sequence and GA for parameter estimation of Muskingum model. Empirical results that involve historical data from existed paper reveal the proposed HCGA outperforms other approaches. The MPSO found the best parameter values compared to previous results. Luo and Xie (2010) proposed immune colonial selection algorithm (ICSA). Wang *et al.* (2010) presented a modified particle swarm algorithm (MPSO) for parameter optimization of Muskingum model. Barati (2011) applied the Nelder-Mead simplex (NMS) algorithm for the purpose of estimating the parameters of the nonlinear Muskingum model. Karahan (2012) proposed two easy methods namely “Trial Error with Spreadsheet” and “Nonlinear optimization with Solver” for the determination of Muskingum parameters. Both of the proposed procedures are tested for the three data sets. It has been shown that the model results are in good agreement with the observation values. Karahan *et al.* (2012) proposed a hybrid harmony search (HS) algorithm for the parameter estimation of the nonlinear Muskingum model. Gemm (2014) discussed two questions raised in the parameter estimation optimization for the nonlinear Muskingum flood routing model.

In this paper, an optimal parameter estimation method for the nonlinear Muskingum model, namely artificial bee colony (ABC) algorithm, is proposed. ABC algorithm developed by Karaboga (2005) is an optimization algorithm which mimics the foraging behavior of honey bees. The performance of this method was applied to two case studies in the case study 1 the data set from Wilson (1974) was used. This data set has

also been extensively studied by others (Gill, 1978; Tung, 1985; Yoon and Padmanabhan, 1993; Mohan, 1997; Kim *et al.*, 2001; Geem, 2006; Luo and Xie, 2010; Karahan, 2012; Karahan *et al.*, 2012). Case study 2 was employed based on the inflow and outflow hydrographs from Karoon River, Iran, and the obtained results was compared with those obtained from genetic algorithm (GA). The main objective of our research was demonstrate whether the ABC algorithm could effectively and efficiently be applied to estimate optimal values of coefficients of the nonlinear Muskingum model.

## 2 ARTIFICIAL BEE COLONY (ABC) ALGORITHM

In the ABC algorithm, the colony of artificial bees contains three groups of bees: employed bees, onlookers and scouts. The employed bees go to an already discovered source and carry its nectar to the hive. The onlooker bees and scout bees are unemployed bees. The onlooker bees are watching the dances of the employed bees and choose a food source according to the quality of the source. The scout bees use external clues or internal motivation to find new undiscovered food sources. In the ABC algorithm, first half of the colony consists of employed artificial bees and the second half constitutes the onlookers. For every food source, there is only one employed bee. The employed bee whose food source is exhausted by the employed and onlooker bees becomes a scout.

In the ABC algorithm, each cycle of the search consists of three steps: sending the employed bees onto the food sources and then measuring their nectar amounts; selecting of the food sources by the onlookers after sharing the information of employed bees and determining the nectar amount of the foods; determining the scout bee and then sending them onto possible food sources.

In the ABC algorithm, the position of a food source represents a possible solution of the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. The number of the employed bees or the onlooker bees is equal to the population size.

The main steps of the algorithm are given below (Kisi *et al.*, 2012):

- (1) Initialization
- (2) Evaluate the population

**Repeat**

- (3) Employed Bees Phase
- (4) Onlooker Bees Phase
- (5) Scout Bees Phase
- (6) Memorize the best food source discovered so far

**Until**

Termination

Criteria is satisfied

Step 1: The ABC generates a randomly distributed initial population  $P(G = 0)$  of the number of food sources ( $SN$ ) solutions (food source positions), where  $SN$  denotes the size of population. Each solution (food source)  $x_i$  ( $i = 1, 2, \dots, SN$ ) is a  $D$ -dimensional vector. Here,  $D$  is the number of optimization parameters. In this study,  $D$  is three parameters for the nonlinear form of the Muskingum model.

Step 2: After initialization, the population is evaluated by substituting each solution ( $\bar{x}_i$ ) in the objective function ( $f$ ) which is the mathematical model of the system to be optimized as a minimization problem. The task of optimization process (Eq. (4)) is to find the optimal parameter vector  $(\bar{x}_i)^*$  that minimizes the objective function:

$$(\bar{x}_i)^* = \arg \min \quad (4)$$

By the evaluation, each solution is assigned a cost function ( $f_i = f(\bar{x}_i)$ ) and a fitness value ( $fitness_i$ ) by the following equation:

$$fitness(\bar{x}_i) = \begin{cases} 1/(1 + f(\bar{x}_i)) & f(\bar{x}_i) \geq 0 \\ 1 + |f(\bar{x}_i)| & f(\bar{x}_i) < 0 \end{cases} \quad (5)$$

Step 3: In the employed bees' phase, a local search is conducted in the neighborhood of each solution by the following equation:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \quad (6)$$

where  $k \in \{1, 2, \dots, BN\}$  and  $j \in \{1, 2, \dots, D\}$  are randomly chosen indexes. Although  $k$  is determined randomly, it has to be different from  $i$ .  $\phi_{ij}$  is a random real number coming from uniform distribution within the range  $[-1, 1]$ . It controls the production of a neighbor food source position around  $x_{ij}$  and the modification represents the comparison of the neighbor food positions visually by the bee. After each candidate source position  $v_{ij}$  is produced and then its quality is evaluated by the fitness function given by Eq. (5), its performance was compared with that of  $x_{ij}$ . If the new food has equal or better nectar than the old source, it was replaced with the old one in the memory. Otherwise, the old one was retained. In other words, a greedy selection mechanism was employed as the selection operation between the old and the current food sources. To simulate the negative feedback which counterbalances the positive feedback, a solution may be abandoned during search likewise abandoning an exhausted food source by real bees. ABC algorithm determines the source to be abandoned by checking the number of local searches conducted in the neighborhood

of the solution. Therefore, if  $\bar{x}_i$  is kept in the population, exploitation counter is incremented by 1. If new solution is accepted, that counter is set to 0.

Step 4: When the searches in the neighborhoods of all solutions are completed in the employed bees' phase, information about the sources is given to the onlookers, which is the simulation of the dances in a real hive. Since the onlookers choose more profitable solutions, profitability of the solutions should be measured. The probability of a source is proportional to its relative fitness (quality) with respect to whole population. In ABC algorithm, the probability values are calculated by Eq. (7) and the onlookers are distributed by a roulette-wheel like selection procedure.

$$p_i = \frac{fitness_i}{\sum_{j=1}^{SN} fitness_j} \quad (7)$$

To apply roulette-wheel selection, a random real number within the range [0,1] is drawn for each source, and if  $p_i$  value is bigger than this random number, an onlooker bee is assigned to the chosen source to make a local search in its neighborhood by Eq. (6). Once a neighbor solution,  $\bar{v}_i$ , is produced, it is evaluated. A greedy selection is applied between current solution,  $\bar{x}_i$ , and new solution,  $\bar{v}_i$ , as in employed bees' phase. If  $\bar{v}_i$  is superior to  $\bar{x}_i$ , it is kept in the population. Otherwise,  $\bar{x}_i$  is kept and exploitation counter is incremented by 1. This process is repeated until the total number of local searches is equal to the number of food sources (SN).

Step 5: During the employed bees' and onlooker bees' phases, the number of local searches (number of exploitations) in the neighborhood of the current solution is

incremented if it cannot be improved by the local search. When new solution is kept in the population instead of the current one, the counter associated with the new solution is reset to 0. If the number of local searches exceeds a number which is called "limit", this solution is assumed to be exhausted and it is abandoned. A new randomly produced solution is generated to be replaced with the exhausted one.

Step 6: Solutions assigned cost function and fitness value is subjected to the employed bees', onlooker bees' and scout bees' phases and some better solutions may be found in the search space through the searches performed in these steps. When a cycle is completed, the best solution in whole population is selected and it is stored in a variable. This is memorization task of the algorithm.

Step 7: The search process conducted through the phases of the algorithm is repeated until a termination criteria is satisfied. The termination criteria may be reaching a maximum number of cycles, producing a satisfactory goal value at the output of the system completing a predefined time limit, etc.

The objective of parameter estimation for the nonlinear Muskingum model is to determine a set of optimal values of parameters that will give minimum difference between the observed and routed outflows. To achieve the best estimation of parameter values, the SSQ of differences between the observed and routed values of outflow is minimized. The objective function for the parameters  $K$ ,  $x$ , and  $m$  is written as:

$$\min SSQ = \sum_{i=1}^n (O_i - \bar{O}_i)^2 \quad (8)$$

Or

$$\min SSQ = \sum_{i=1}^n \left\{ O_i - \left[ \left( \frac{1}{1-x} \right) \left( \frac{S_i}{K} \right)^{\frac{1}{m}} - \left( \frac{x}{1-x} \right) I_i \right] \right\}^2 \quad (9)$$

where  $\bar{O}_t$  is routed outflow at time  $t$ .

In this study, the ABC algorithm written in MATLAB software was employed for nonlinear form of Muskingum model. There were three real variables  $K$ ,  $x$ , and  $m$  that need to be estimated.

### 3 PERFORMANCE EVALUATION

Four different types of standard statistical were considered as statistical performance evaluation. The sum of the square of the deviations between the computed and observed outflows (SSQ), the sum of the absolute value of the deviations between the computed and observed outflows coefficients (SAD), the mean absolute errors between the routed and observed outflows (MAE) and the mean relative errors between the routed and observed outflows (MRE) were used. The four performance evaluation criteria used in this study can be calculated utilizing the following equations:

$$SSQ = \sum_{i=1}^n (O_t - \bar{O}_t)^2 \quad (10)$$

$$SAD = \sum_{i=1}^n |O_t - \bar{O}_t| \quad (11)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |O_t - \bar{O}_t| \quad (12)$$

$$MRE = \frac{1}{n} \sum_{i=1}^n \left| \frac{O_t - \bar{O}_t}{O_t} \right| \times 100 \quad (13)$$

where  $O_t$  and  $\bar{O}_t$  are, respectively the observed and routed outflows and  $n$  is the number of data.

## 4 APPLICATION AND RESULTS

### 4.1 Case study 1

To investigate the performance of the ABC technique for the parameter estimation of the nonlinear Muskingum model, this technique was applied to an example first proposed by Wilson (1974). The reasons for selecting this example are as follows: (1) This example presents a pronounced nonlinear relationship between weighted flow and storage volume (Yoon and Padmanabhan, 1993; Mohan, 1997) (2) this example has also been used by Gill (1978), Tung (1985), Yoon and Padmanabhan (1993), Mohan (1997), Kim *et al.* (2001), Das (2004), Geem (2006), Luo and Xie (2010), Karahan (2012) and Karahan *et al.* (2012) for testing different parameter estimation methodologies. Therefore, the performance of the proposed ABC can effectively be compared with the previous reported results obtained with this example.

After a thorough study of the sensitivity of the ABC algorithm parameters on the resulting errors in a population size of 500, a scout bee production of moderate value (Limit =  $n_e \times D$ ) and an evolution generation (EG) of 200 were used in the ABC algorithm for the estimation of the nonlinear Muskingum model parameters. The lower and upper bound of  $K$ ,  $x$ , and  $m$  parameters were used (0.01, 0.4), (0.2, 0.3) and (1.5, 2.5). Since ABC algorithm is initiated randomly, the problem was solved for 20 different random seeds and the best (minimum) out of 20 runs was taken as the solution of the problem.

The routed outflow using the ABC method presented in Table 1. Table 1 also shows the outflows obtained using the least-squares method (LSM) (Gill, 1978), the HJ pattern search in conjunction with the CG method (HJ+CG), and HJ with the DFP method (HJ+DFP) (Tung, 1985), NONLR (Yoon and Padmanabhan, 1993), GA (Mohan, 1997), HS (Kim *et al.*, 2001), BFGS (Geem, 2006), and

ICSA method (Luo and Xie, 2010), and “Trial-Error with Spreadsheet” and “Nonlinear Optimization with Solver” (Karahan, 2012). The routed outflow hydrographs along with the observed inflow and outflow hydrographs are

also presented in Figure 1. It is obvious from Figure 1 that the routed outflow hydrographs using ABC and the other methods, except Gill’s (1978) method, closely follow the observed outflow hydrograph.

**Table 1** Observed and computed values of outflow for data set given by Wilson (1974)

Time (h)	Inflow ( $\text{m}^3 \text{s}^{-1}$ )	Observed outflow ( $\text{m}^3 \text{s}^{-1}$ )	Routed outflow ( $\text{m}^3 \text{s}^{-1}$ )											
			LSM	HJ+C G	HJ+DF P	NONL R	GA	HS	BFG S	ICSA	Trial-Error	Solver	ABC	
0	22	22	22	22	22	22	22	22	22	22	22	22	22	22
6	23	21	22	22	22	22.6	22	22	22	22	21.86	21.87	22	
12	35	21	22.8	22.4	22.4	23	22.4	22.4	22.4	22.4	20.45	20.52	22.4	
18	71	26	29.6	26.8	26.7	24.2	26.3	26.6	26.6	26.6	18.76	19.07	26.7	
24	103	34	39.1	34.9	34.8	33.2	34.2	33.4	34.5	34.4	26.3	26.9	34.6	
30	111	44	47.6	44.5	44.7	47.1	44.2	44.1	44.2	44.2	42.81	43.58	44.2	
36	109	55	58	56.7	56.9	56.8	56.9	56.8	56.9	56.9	58.76	59.58	56.8	
42	100	66	67.1	67.3	67.7	66.2	68.2	68.1	68.1	68.1	71.57	72.32	67.9	
48	86	75	74.8	75.9	76.3	75	77.1	77.1	77.1	77.1	80.06	80.65	76.9	
54	71	82	80.4	81.9	82.2	80.7	83.2	83.3	83.3	83.3	83.52	83.91	83.2	
60	59	85	83.2	84.5	84.7	83.5	85.7	85.9	85.9	85.9	82.32	82.51	85.8	
66	47	84	82.8	83.4	83.5	84.3	84.2	84.5	84.5	84.5	78.64	78.63	84.5	
72	39	80	80.1	79.9	79.8	79.9	80.2	80.6	80.6	80.5	72.48	72.32	80.6	
78	32	73	74.5	73.6	73.3	74.3	73.3	73.7	73.7	73.6	65.76	65.49	73.8	
84	28	64	67.2	65.8	65.5	65.3	65.5	65.4	65.4	65.3	58.56	58.21	65.5	
90	24	54	58.1	56.9	56.5	55.9	55.8	56	56	55.9	52.1	51.7	56.1	
96	22	44	48.1	47.8	47.5	45.1	46.7	46.7	46.7	46.6	45.92	45.5	46.7	
102	21	36	37.6	38.9	38.7	35.4	38	37.8	37.8	37.7	40.56	40.15	37.7	
108	20	30	28.2	31.5	31.4	28.7	30.9	30.5	30.5	30.5	36.2	35.82	30.4	
114	19	25	21.9	25.8	25.9	24.3	25.7	25.3	25.2	25.3	32.62	32.26	25.1	
120	19	22	19.1	22	22.1	20.9	22.1	21.8	21.7	21.8	29.49	29.17	21.6	
126	18	19	19	20.1	20.2	20.4	20.2	20	20	20	27.22	26.93	19.9	

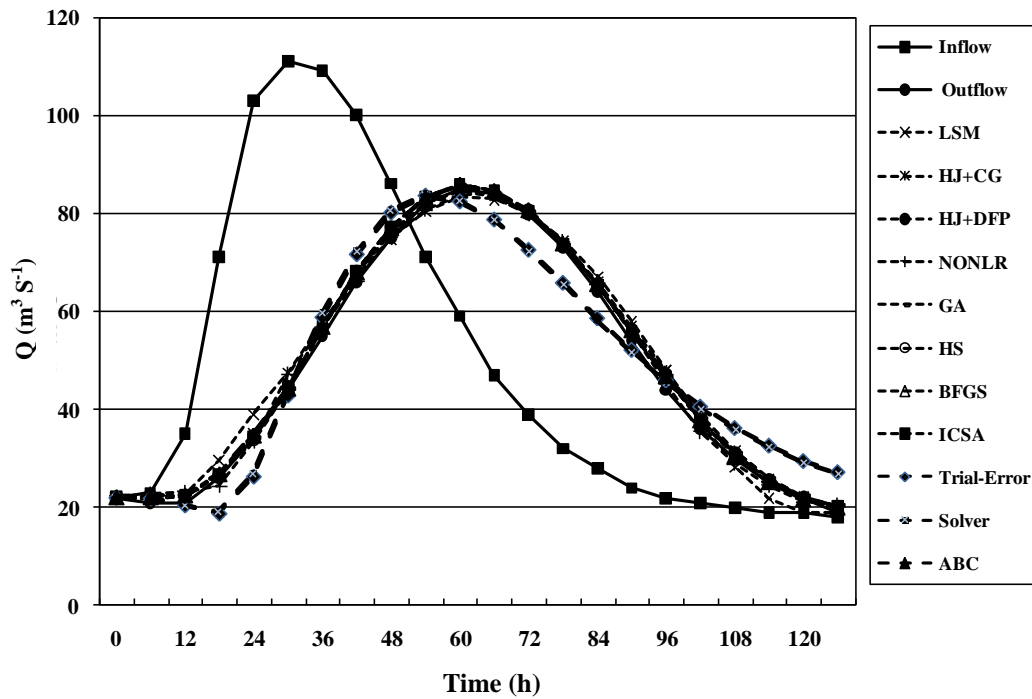


Figure 1 Inflow hydrograph, outflow and routed hydrographs with different methods

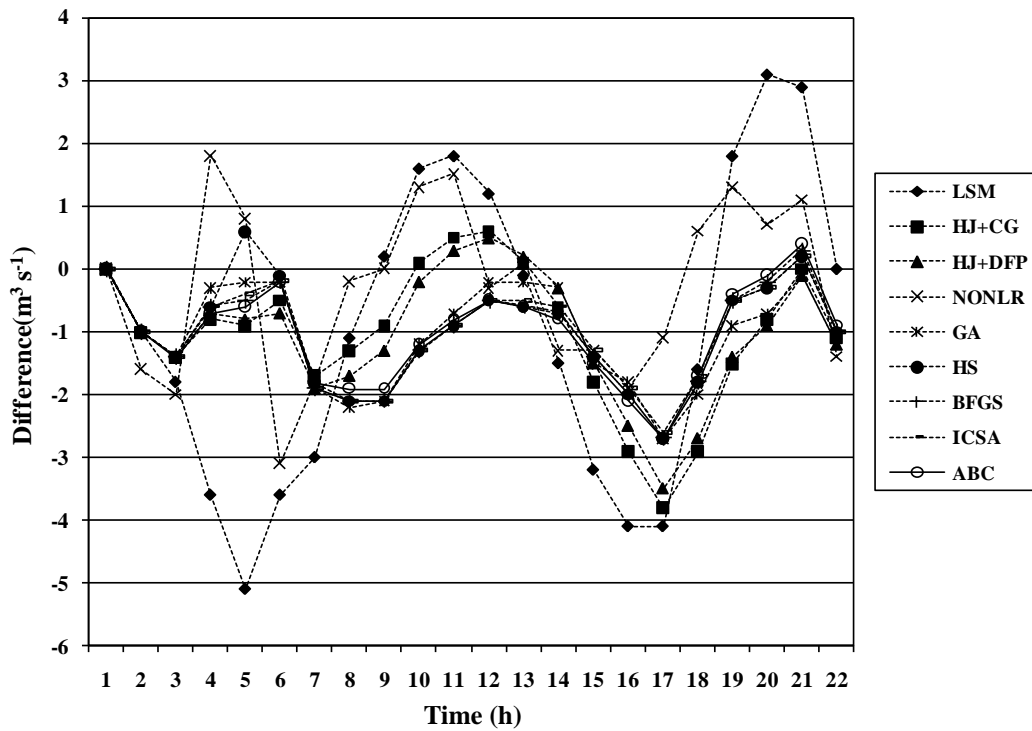


Figure 2 Difference between the routed and observed outflows



The differences between the observed and routed outflows of these eleven methods are presented in Figure 2.

The parameters estimated by the ABC, Trial-Error, Solver, ICSA, GA, BFGS, HS, NONLR, HJ+DFP, HJ+CG, and LSM are presented in Table 2. For floods, the peak outflow and its time of occurrence are the important factors in assessing the impacts and controlling them. The deviations of peak of routed and actual outflows (DPO) and deviation of peak time of routed and actual outflows (DPOT) evaluate how the amount and time of occurrence of peak flow, respectively, are predicted. The values of these performance measures are also given in Table 2.

By studying Table 2, we can see that the estimated value of the weighting factor  $x$  differs very little among the various methods, but the values of  $K$  and  $m$  vary quite significantly. ABC algorithm employed in this study has a balanced exploration and exploitation capability and therefore it can escape from local minima of multimodal surfaces. Also, the results achieved with ABC is satisfactory in computational precision with  $SSQ=35.62$ ,  $SAD=23.2$ ,  $MAE=1.05$  and  $MRE=2.49$ . Compared with those eleven methods, ABC model was found to reduce the  $SSQ$  by 0.056, 3.23, 3.26, 7.32, 21.44, 27.84, 39.35, 309.01 %, 1612.46 % and 1600.36 % as

compared to the ICSA, GA, BFGS, HS, NONLR, HJ+DFP, HJ+CG, LSM, Trial-Error, and Solver algorithms, respectively. It is clear from those values that the performance of the ABC for parameter optimal estimation of the nonlinear Muskingum model is better than the GA, BFGS, HS, NONLR, HJ+DFP, HJ+CG, LSM, Trial-Error, and Solver algorithms and as good as the ICSA algorithm.

It is clear from DPO and DPOT values that the performance of the ABC algorithm is less than the (HJ + CG), (HJ + OFP), NONLR, and GA methods, and it definitely resulted in better predicted values of outflows than the LSM, Trial-Error, Solver, HS, BFGS, and ICSA. It may be noted that even though the NONLR method resulted in comparatively good performance, the routed flows are shifted by one time unit (6 h) for the peak flow.

To illustrate the precision and speed of the ABC method for parameter optimal estimation of the nonlinear Muskingum model, the estimating procedures were performed using a different evolution generation (EG) as the termination condition for the ABC technique. Table 3 shows the hydrologic parameter values that were obtained from the ABC method. Table 3 also presented the corresponding  $SSQ$  and  $SAD$  values that were calculated from the observed and routed outflows.

**Table 2** Comparison of the parameter values and statistics of different methods for data set given by Wilson (1974)

Method	K	x	m	SSQ	SAD ( $m^3 s^{-1}$ )	MAE ( $m^3 s^{-1}$ )	MRE (%)	DPO	DPOT
LSM	0.01	0.25	2.347	145.69	46.26	2.1	5.61	1.80	0
HJ+CG	0.0669	0.2685	1.9291	49.64	25.2	1.15	2.99	0.50	0
HJ+DFP	0.0764	0.2677	1.8978	45.54	24.8	1.13	2.96	0.30	0
NONLR	0.06	0.27	2.36	43.26	25.2	1.15	3.27	0.70	1
GA	0.1033	0.2813	1.8282	38.23	23	1.05	2.53	0.70	0
HS	0.0833	0.2873	1.863	36.78	23.41	1.06	2.55	0.90	0
BFGS	0.0863	0.2869	1.8679	36.77	23.46	1.07	2.55	0.90	0
ICSA	0.0884	0.2862	1.8624	35.64	23	1.05	2.49	0.90	0
ABC	0.3180	0.2869	1.8863	35.62	23.2	1.05	2.49	0.80	0

**Table 3** Sensitivity analysis of the parameter values in terms of the number of evolution generations for data set given by Wilson (1974)

Evolution Generation (EG)	$K$	$x$	$m$	SSQ	SAD(m <sup>3</sup> s <sup>-1</sup> )
00	0.318064	0.286979	1.886372	35.62	23.2
300	0.318064	0.286979	1.886372	35.62	23.2
200	0.318064	0.286979	1.886372	35.62	23.2
100	0.348949	0.286041	1.865179	35.73	23.1
50	0.337298	0.287105	1.873139	35.88	23.2
20	0.336275	0.284609	1.873071	36.15	23.5
10	0.339568	0.286113	1.871768	36.65	23.3

It is commonly desired that a method as an estimator should be unbiased, which means that the mean difference should be zero, and as such there should be both positive and negative differences. From Figure 2, it can be seen that the gradient-type methods such as NONLR and LSM show differences that are both positive and negative, and the mean difference that might be close to zero.

However, the remaining methods, including the ABC, which are all near optimal methods, show differences that are almost all positive. This undesirable bias should be analyzed and discussed. In order to illustrate the reason of this bias, we must first prove that if the ABC method is biased or unbiased. Supposed  $K=0.3180$ ,  $x=0.2869$ , and  $m=1.8863$ , the outflows can be generated by Eq. (3) and presented in Table 4. The parameter estimations were carried out, respectively, using a different EG as the termination condition for the ABC approach. The routed outflows obtained from the ABC and the corresponding differences calculated from the generated and routed outflows are presented in Table 4. Figure 3 shows the

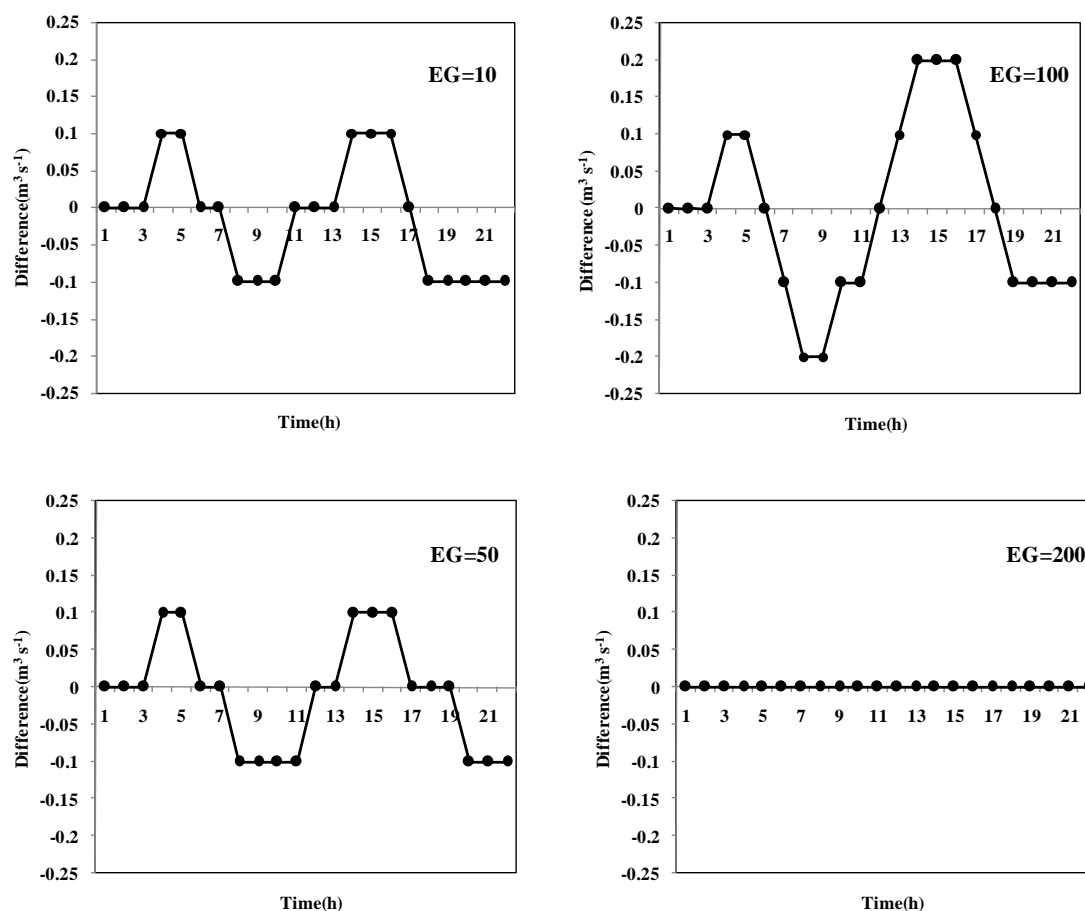
corresponding differences in evolutionary generation ranges from 10 to 200. It is clear that the differences are both positive and negative, and the mean differences close or equal to zero. As a result, Figure 3 suggests that the ABC method is unbiased.

On the other hand, although Figure 2 shows that the NONLR method and the LSM are unbiased, but the SSQs from the NONLR method and the LSM are much greater than the SSQ from the ABC approach. While we set the SSQ of the ABC method at the same level with the NONLR method and the LSM, respectively, the differences between the observed and routed outflows are both positive and negative and the mean differences close to zero separately. Therefore, we can conclude that the undesirable bias is derived from the error of observed outflows not the ABC method itself.

In order to analyze the Muskingum of the ABC algorithm, it has been run with different population sizes (colony sizes) and evolutionary generation ranges from 10 to 1000. In Table 5, the SSQ values with different colony sizes varying as 25 to 1000 have been presented.

**Table 4** Comparison of the generated and routed outflows for data set given by Wilson (1974)

Time (h)	Inflow (m <sup>3</sup> s <sup>-1</sup> )	Generated Outflow (m <sup>3</sup> s <sup>-1</sup> )	Routed Outflow (m <sup>3</sup> s <sup>-1</sup> )					Differences (m <sup>3</sup> s <sup>-1</sup> )				
			Evolution Generation (EG)									
			10	20	50	100	200	10	20	50	100	200
0	22	22	22	22	22	22	22	0	0	0	0	0
6	23	22	22	22	22	22	22	0	0	0	0	0
12	35	22.4	22.4	22.4	22.4	22.4	22.4	0	0	0	0	0
18	71	26.7	26.6	26.7	26.6	26.6	26.7	0.1	0	0.1	0.1	0
24	103	34.6	34.5	34.6	34.5	34.5	34.6	0.1	0	0.1	0.1	0
30	111	44.2	44.2	44.4	44.2	44.2	44.2	0	-0.2	0	0	0
36	109	56.8	56.8	57	56.8	56.9	56.8	0	-0.2	0	-0.1	0
42	100	67.9	68	68.2	68	68.1	67.9	-0.1	-0.3	-0.1	-0.2	0
48	86	76.9	77	77.1	77	77.1	76.9	-0.1	-0.2	-0.1	-0.2	0
54	71	83.2	83.3	83.3	83.3	83.3	83.2	-0.1	-0.1	-0.1	-0.1	0
60	59	85.8	85.8	85.8	85.9	85.9	85.8	0	0	-0.1	-0.1	0
66	47	84.5	84.5	84.5	84.5	84.5	84.5	0	0	0	0	0
72	39	80.6	80.6	80.5	80.6	80.5	80.6	0	0.1	0	0.1	0
78	32	73.8	73.7	73.6	73.7	73.6	73.8	0.1	0.2	0.1	0.2	0
84	28	65.5	65.4	65.3	65.4	65.3	65.5	0.1	0.2	0.1	0.2	0
90	24	56.1	56	55.9	56	55.9	56.1	0.1	0.2	0.1	0.2	0
96	22	46.7	46.7	46.6	46.7	46.6	46.7	0	0.1	0	0.1	0
102	21	37.7	37.8	37.7	37.7	37.7	37.7	-0.1	0	0	0	0
108	20	30.4	30.5	30.4	30.4	30.5	30.4	-0.1	0	0	-0.1	0
114	19	25.1	25.2	25.2	25.2	25.2	25.1	-0.1	-0.1	-0.1	-0.1	0
120	19	21.6	21.7	21.7	21.7	21.7	21.6	-0.1	-0.1	-0.1	-0.1	0
126	18	19.9	20	20	20	20	19.9	-0.1	-0.1	-0.1	-0.1	0



**Figure 3** Difference between the routed and generated outflows in evolutionary generation ranges from 10 to 200

**Table 5** Sensitivity analysis of the parameter values in terms of the number of evolution generations under different colony sizes for data set given by Wilson (1974)

Evolution Generation (EG)	Colony size					
	25	50	100	200	500	1000
1000	35.82	35.72	35.82	35.64	35.62	35.68
500	35.82	35.72	36.1	35.64	35.62	35.68
300	35.82	35.72	36.37	35.64	35.62	35.68
200	35.9	35.72	36.37	35.64	35.62	35.81
100	38.01	36.07	36.48	35.64	35.62	35.92
50	39.48	36.55	36.48	35.99	35.73	36.13
20	47.68	38.52	37.02	37	36.26	36.27
10	49.2	39.61	37.68	37.77	36.26	36.36

By studying Table 5, it is obvious that as the population size increases, the algorithm produces generally better results. However, after a sufficient value for colony size, any increment in the value does not improve the performance of the ABC algorithm significantly. For the test problems carried out in this work, the colony size of 200-500 can provide an acceptable convergence speed for search.

As mentioned before, the “scout bee” production is controlled by the control parameter “limit” in the ABC algorithm. There is an inverse proportionality between the value of “limit” and the scout production frequency. As the value of “limit” approaches to infinity, the total number of the scouts produced goes to zero. The results of the ABC algorithm presented in Table 2 were obtained for the colony sizes of 100, 200 and 500. In order to show the effect of the scout production process on the performance of the algorithm, the SSQ values found for the different “limit” values ( $0.1 \times n_e \times D$ ,  $0.5 \times n_e \times D$ ,  $n_e \times D$  and “without scout”) and colony sizes (100, 200 and 500) is presented in Table 6.

As seen from Table 6, for colony size 100, when the scout production frequency is very high (limit value =  $0.1 \times n_e \times D$ ) or zero (without scout), the results obtained by the ABC algorithm are worse than those produced by using the moderate values for limit, such as,  $0.5 \times n_e \times D$  and  $n_e \times D$ . For colony sizes 200 and 500, the production of scouts does not have any useful effect on the performance of the algorithm. However, as expected, it improves the search ability of the algorithm for the nonlinear Muskingum model and its benefit becomes much clearer for the smaller colony sizes.

## 4.2 Case study 2

The Karoon River basin, with a basin area of 67,000 km<sup>2</sup>, is located in southern part of Iran between longitudes 48° 15' and 52° 30' east, latitude 30° 17' and 33° 49' north. The origin of the Karoon River is 75 km far from the south of Esfahan city in the Zagros Mountain ranges and divided in two branches, Gargar and Shatit, in the north of Shooshtar city, in Bandeghir the two branches and the Dez river join each other and form a great river called the Karoon. The reach of between stations of Molasani (station no: 21-308, 48°53' E, 31°35'N) as an upstream station and Ahwaz (station no:21-309, 48°40'E, 31°20'N) as a downstream station on the Karoon River operated by Khuzestan Water and Power Authority is treated in this study. Flood data from 30/11/2008 to 3/12/2008 are considered for the purpose. In the genetic algorithm, a population size of 100, a crossover probability of 0.9 and a mutation probability of 0.001 were used for the estimation of the nonlinear Muskingum model parameters (Mohan, 1997). In ABC algorithm, a population size of 500, a scout bee production of moderate value (Limit =  $n_e \times D$ ) and a EG of 200 were used for the estimation of the nonlinear Muskingum model parameters. Since genetic and ABC algorithms are initiated randomly, the problem was solved for 20 different random seeds and the best (minimum) out of 20 runs was taken as the solution of the problem. The lower and upper bound of  $K$ ,  $x$ , and  $m$  parameters were used (1, 100), (0.000001, 0.3) and (0.01, 1). The routed outflow hydrographs along with the observed inflow and outflow hydrographs are presented in Figure 4 and Table 7.

The parameters estimated by the ABC and GA are presented in Table 8.

**Table 6** Effect of the ‘‘limit’’ value, which controls the scout production, on the performance of the ABC algorithm (The bold value indicates the best among the values obtained under different limit values)

Colony size	100 (ne = no = 50)				200 (ne = no = 100)				500 (ne = no = 250)			
	Limit			Without scout	Limit			Without scout	Limit			Without scout
	0.1×ne×D	0.5×ne×D	ne×D		0.1×ne×D	0.5×ne×D	ne×D		0.1×ne×D	0.5×ne×D	ne×D	
Mean	36.163	35.983	35.982	36.068	35.923	35.974	35.985	35.935	35.781	35.806	35.804	35.828
S.D.	0.1969	0.1358	0.1967	0.1931	0.1355	0.1878	0.1845	0.1794	0.0811	0.1238	0.0958	0.1276

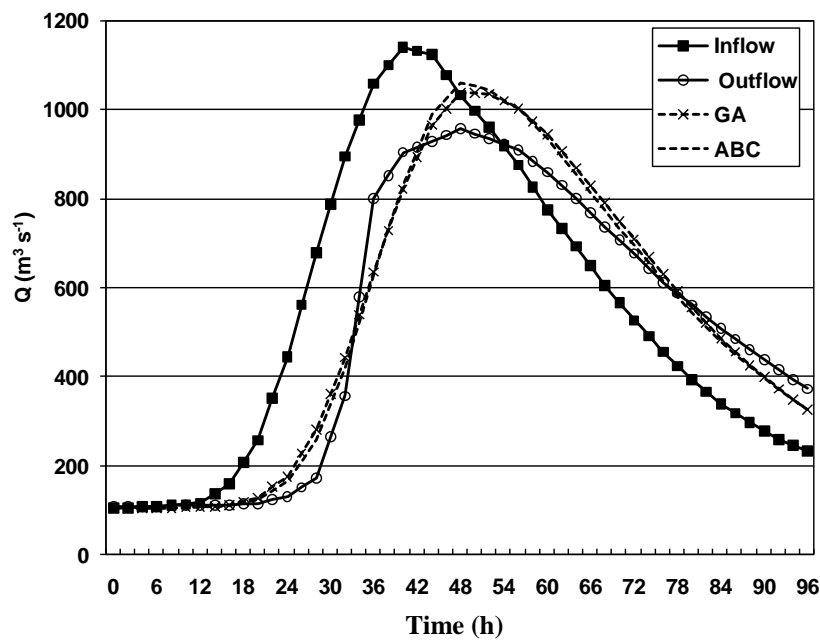
ne, Number of employed bees; D, dimension of the problem; runs =120; total evaluation number = 15,000

**Table 7** Observed and computed values of outflow for the Karoon River data set

Time (h)	Inflow (m <sup>3</sup> s <sup>-1</sup> )	Observed outflow (m <sup>3</sup> s <sup>-1</sup> )	Routed outflow (m <sup>3</sup> s <sup>-1</sup> )	
			GA	ABC
0	105	108	105	105
2	106.5	108.5	105	105
4	108	109	105	105
6	109.5	109.5	105.5	105.4
8	111	110	106.1	105.8
10	114	111	106.9	106.5
12	117	112	107.8	107.3
14	138.5	112	109.4	108.6
16	160	112	111.1	110
18	209	114	119.9	117.1
20	258	116	128.8	124.2
22	352	123.5	152.7	144.3
24	446	131	176.6	164.4
26	562.5	152	228.7	211.6
28	679	173	280.8	258.8
30	787	265	361.7	339.3
32	895	357	442.7	419.9
34	977	578.5	538.8	523.1
36	1059	800	634.9	626.2
38	1100	852.5	728	729.6
40	1141	905	821.1	833.1
42	1132.5	916.5	892.9	911.6
44	1124	928	964.7	990.1
46	1078.5	943	1000.9	1025.4
48	1033	958	1037.1	1060.8
50	997.5	946	1036.1	1053.4
52	962	934	1035.2	1046
54	919	921.5	1018.5	1023.8

**Table 7** (Continue)

56	876	909	1001.9	1001.7
58	825.5	884.5	973.3	969.1
60	775	860	944.8	936.4
62	734	830.5	906.6	895.4
64	693	801	868.4	854.5
66	649.5	769	829.3	814.6
68	606	737	790.3	774.7
70	566.5	707	749.7	734.2
72	527	677	709.1	693.8
74	491.5	644.5	669.5	655.1
76	456	612	629.9	616.4
78	424.5	587	592.5	580.4
80	393	562	555.2	544.5
82	366.5	535.5	520.8	511.7
84	340	509	486.5	479
86	318.5	485	455.9	450
88	297	461	425.4	421
90	278	438	399	396.1
92	259	415	372.6	371.2
94	246	394	349.5	349.4
96	233	373	326.5	327.7

**Figure 4** Inflow hydrograph, outflow and routed hydrographs with GA and ABC algorithms

**Table 8** Comparison of the parameter values and statistics of GA and ABC algorithms for the Karoon River data set

Method	K	x	m	SSQ	SAD ( $\text{m}^3 \text{s}^{-1}$ )	MAE ( $\text{m}^3 \text{s}^{-1}$ )	MRE (%)	DPO	DP OT
GA	20.92	0.00001	0.8913	182821.1	2317.35	47.29	9.41	79.1	0
ABC	76.94	0.0001	0.7143	177161.4	2222.25	45.35	8.83	102.8	0

It is obvious from Table 6 that the ABC performs better than the GA with  $\text{SSQ}=177161.4$ ,  $\text{SAD}=2222.25$ ,  $\text{MAE}=45.35$  and  $\text{MRE}=8.83$ . ABC algorithm was found to reduce the SSQ by 3.09% as compared to the GA. The performance of the ABC algorithm is less than the GA algorithm in respect to DPO and as good as the GA in respect to DPOT.

## 5 CONCLUSION

The task of parameter estimation in the calibration process of the Muskingum model becomes more involved if the routing equations are nonlinear. In the past, various parameter estimation methods have been proposed to determine the parameter values for the nonlinear Muskingum model. In this paper, the ABC algorithm is developed for the parameter optimization of Muskingum models. The accuracy of the ABC was compared with the other reported methods based on the different criteria. The results indicated that the ABC approach generally performed better than the Trial-Error, Solver, LSM, HJ+CG, HJ+DFP, NONLR, GA, HS, BFGS, and ICSA methods for parameter estimation of the nonlinear Muskingum model. Comparison of the Trial-Error, Solver, LSM, HJ+CG, HJ+DFP, NONLR, HS, BFGS, ICSA, and ABC methods in respect to sum of the square of the deviations between the routed and observed outflows (SSQ) indicated that the ABC performed better than other methods and as good as the ICSA. On the other hand, according to the sum of the absolute value of the deviations (SAD), the mean absolute errors (MAE) and the mean relative errors (MRE) between the routed and observed

outflows, the ABC was found to perform as good as the GA and ICSA methods and better than the Trial-Error, Solver, LSM, HJ+CG, HJ+DFP, NONLR, HS, and BFGS methods. According to the comparison of routed and observed outflow hydrographs' peak and time to peak, the ABC was found to perform better than LSM, Trial-Error, Solver, NONLR, HS, BFGS, and ICSA methods and less than HJ+CG, HJ+DFP, and GA methods with  $\text{DPO}=0.8$  and  $\text{DPOT}=0$ .

The GA and ABC algorithms were compared for the Karoon River data set. The results showed that the ABC performs better than the GA in respect to SSQ, SAD, MAE and MRE values. According to DPO and DPOT, the ABC was found to perform less than GA with  $\text{DPO}=102.8$  and  $\text{DPOT}=0$ . The results of the study are highly encouraging and suggest that ABC algorithm is viable for parameter estimation of the nonlinear Muskingum model.

In this study, ABC algorithm was used for the parameter optimization of Muskingum model. Other optimization techniques such as Ant Colony and Tabu Search may also be used for the parameter optimization of Muskingum model instead of ABC and their accuracies may be compared with each other.

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## تخمین بهینه پارامترهای مدل ماسکینگام غیرخطی بر اساس الگوریتم کلونی زنبور عسل

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**چکیده** تخمین پارامترهای مدل ماسکینگام غیرخطی مسئله کاملاً غیرخطی است. اگرچه روش‌های متعددی برای بهینه‌سازی ضرایب مدل روندیابی سیلاب ماسکینگام غیرخطی به کار گرفته شده است اما هنوز روش کارآمدی برای این هدف ارائه نشده است. در این مقاله صحت الگوریتم کلونی زنبورعسل برای بهینه‌سازی پارامترهای ماسکینگام غیرخطی بررسی شده و کارایی این الگوریتم با روش‌های دیگر بهینه‌سازی مقایسه شد. برای ارزیابی توانایی الگوریتم کلونی زنبورعسل، در مطالعه حاضر از تعدادی از نمایه‌های آماری از قبیل ریشه میانگین مربعات خطا، مجموع خطای مطلق، متوسط خطای مطلق و متوسط خطای نسبی استفاده شد. الگوریتم کلونی زنبور عسل الگوریتم هوشمندی است که می‌تواند به طور موثری با سرعت هم‌گرایی زود هنگام و آهسته نسبت به الگوریتم‌های تکاملی سنتی غلبه نماید. بهترین مقادیر پارامترها از نظر مجموع مربعات باقیمانده بین دبی مشاهداتی و روندیابی شده تعیین شد. نتایج شبیه‌سازی نشان داد که کارایی الگوریتم کلونی زنبورعسل با مجموع مربعات انحراف بین دبی محاسباتی و مشاهداتی برابر  $۳۵/۶۲$  مترمکعب بر ثانیه، مجموع مقدار مطلق انحراف بین دبی محاسباتی و مشاهداتی برابر  $۲۳/۲$  مترمکعب بر ثانیه، متوسط خطای مطلق بین دبی محاسباتی و مشاهداتی برابر  $۱/۰۵$  مترمکعب بر ثانیه و متوسط خطای نسبی بین دبی محاسباتی و مشاهداتی برابر  $۲/۹$  درصد قابل مقایسه با دیگر الگوریتم‌هاست. بنابراین روشی کارآ برای بهینه‌سازی پارامترهای مدل ماسکینگام غیرخطی می‌باشد.

**کلمات کلیدی:** برآورد پارامتر، بهینه‌سازی پارامتر، رودخانه کارون، روندیابی سیل، مدل ماسکینگام